LEGO LIGO

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Abstract

Anyone who knows the term interferometer usually thinks of a complex and very precise device that creates very specific light patterns. This idea also includes a heavy, springequipped, and most importantly very expensive optical table with completely precise optical components to achieve maximum control over the entire apparatus. However, very acceptable results can be achieved even with a significantly more primitive design. Our project tries to show not only how the interferometer itself works but also how to make it at home or at school.

From Ether to Black Holes

The origin of the interferometer

At the end of the 19th century, Maxwell's equations of electromagnetism, describing the propagation of light as just a narrow region of electromagnetic waves, slowly became known. But with that arose the question of what medium it propagates through, what is actually waving. Just as a water surface ripples or a sound needs air to propagate, according to contemporary ideas, there had to be a medium for light to propagate. This hypothetical medium was called ether. Another problem was the speed of electromagnetic waves, i.e. the speed of light. That the speed of light is finite has been known since the 17th century. However, with respect to which system does the speed of light have this value which is based on Maxwell's equations and had been measured relatively accurately several times. There was a completely natural idea that this speed of electromagnetic waves is just the speed with respect to the ether (i.e. with respect to some kind of absolute system). With respect to the ether, light should propagate in all directions with the same speed, regardless of whether the source is at rest or in motion with respect to the ether. If we measure the speed of light with respect to a moving source (e.g. with respect to the Earth), we should get different results according to the classical addition of velocities.

Albert Michelson decided to verify this hypothesis (to measure the speed of the Earth with respect to the system connected to the ether). In 1881, he created the first version of an interferometer in Potsdam, now known as the Michelson interferometer, with which he performed a series of experiments. However, this version had a number of shortcomings. The second version and a series of experiments conducted in Cleveland in 1887 with his coworker Morley (the so-called Michelson-Morley experiment [1]) were finally convincing. Michelson-type experiments have been repeated many times, and the sensitivity has been increased enough to measure the Earth's velocity with respect to the ether to within meters per second. However, all results were negative (the measured speed of light was the same in each direction) and did not confirm the ether hypothesis from the end of the 19th century. These (but also other) experiments were satisfactorily explained only by the special theory of relativity.

Use of interferometers

The interferometer gradually found its way into a series of physics experiments and devices. These include devices for measuring surface topography, or large observatories observing cosmic electromagnetic radiation of various wavelengths, as well as other types of waves. Perhaps the most well-known use is the LIGO [2] (Laser Interferometer Gravitational Wave Observatory) experiment to detect gravitational waves, which is very similar to the original Michelson interferometer with its cross-shape. In 2015, LIGO measured the first very strong gravitational waves coming from the merger of a pair of black holes, and thus confirmed the predictions of general relativity about this phenomenon. This success was followed by a number of other successful measurements. The measured gravitational waves from the merger of a pair of black holes were converted into sound, and the video with this alien "bubbling" became relatively well-known [3].

Michelson interferometer

Interference

Before we get to the interferometer itself, let us recall the phenomenon on which the whole interferometer is based and from which it got its name – the interference. Interference (composition) occurs when two waves meet at some point. These are then composed according to the principle of superposition. If they are in phase with each other, constructive interference occurs, and they amplify each other. However, if the phases are opposite (the phase shift is 180°), the destructive interference occurs and the waves superpose to form a resultant wave of a lower amplitude at a given point, or (at the same amplitudes) completely cancel each other out (Fig. 1). Of course, the phase shift can take on different values, producing zero-amplitude waves up to the sum of the amplitudes of the original waves.



Figure 1. Constructive interference of waves in phase with each other on the left, destructive interference of waves with the phase shift of 180° on the right. Source: [4]

Interferometer

The Michelson interferometer is really probably the simplest (and the best known) version of the interferometer ever. This is already evidenced by the fact that only five components are actually enough for its operation: a source of monochromatic coherent light (e.g. laser), a semi-transparent mirror (also known in this context as a beam splitter), two ordinary mirrors, and a detector. The interferometer then has the shape of a cross with a beam splitter in the middle and the other components on its arms (Fig. 2). Anything from a screen on which the resulting beam is projected to sophisticated detection devices can serve as a detector on the fourth arm of the cross.

The beam of light from the source is divided by a semi-transparent mirror into two arms of the interferometer. In each arm, the beam of light bounces off the mirrors back into the beam splitter, which splits the beams again and sends half of each to the detector, while the other returns to the laser.



Figure 2. Simplified diagram of a Michelson interferometer.

The resulting two combined beams then interfere. Since the beams emanate from the source in phase, but each of them passes through a different arm of the interferometer, their resulting phase shift will depend on the lengths of the individual arms, or more precisely on the distance between the mirrors and the beam splitter (Fig. 3). It is worth to notice that the change in the length of one arm has a twofold effect on the final phase shift, as the light beam travels in a given way twice – back and forth. Thus, completely constructive interference occurs when there is a path difference (the difference in the distance travelled by each beam) in multiples of half-wavelength of the light used. The resulting interference pattern on the detector will be a dot, the brightness of which will depend on the lengths of the individual arms of the interferometer.



Figure 3. Phase shift of interfering resulting beams depending on arm lengths (constructive interference on the left, destructive interference on the right). Source: [5]

Interference Patterns

Formation of patterns

Theoretically, the simplest pattern that can be created on the detector is the already mentioned dot. This is based on the assumption that the source beam has a completely cylindrical shape. Unfortunately, this view is unrealistic because the laser beam diverges into a conical shape. The interferometer thus creates a number of different patterns from concentric circles through hyperbolas to parallel lines (Fig. 4). Normally, a diverging lens is added to the interferometer behind the source to intentionally emphasize these patterns.



Figure 4. Two interference patterns created by the interferometer (parallel lines and concentric circles). Source: [6]

To explain the patterns, it is first appropriate to simplify the scheme of the interferometer even more, or rather "straighten" (Fig. 5). If we look from the detector to the beam splitter, we see directly the first of the mirrors M_1 and a reflected image M'_2 of the second mirror M_2 . The patterns can then be interpreted as the result of interference between light coming from the two virtual images S'_1 and S'_2 of the original source S.

If the mirrors are at right angles to each other (Fig. 5a), the resulting pattern will be concentric circles (unless both arms have exactly the same length, in this exceptional case there would be only constructive interference in the whole pattern and the pattern would be bright). If the mirrors are not placed at right angles to each other (Fig. 5b), hyperbolas or, if the arm lengths are identical, seemingly parallel lines occur (in fact, these are again hyperbolic shapes, but their curvature is completely imperceptible).



Figure 5. "Straightened" diagram of the interferometer. Source: [7]

The above-mentioned concentric circles are particularly useful for homemade interferometers on which, with a sufficiently stable design, changes in the lengths of the individual arms can be well observed. The circles either shrink, disappear in the middle while new ones appear at the edge, or increase from the centre. With each change in the path of one beam by one wavelength (and thus with a change in the length of the arm by a half-wavelength), the pattern shifts by one circle and apparently returns to its original form [8]. Conversely, this fact can be used to calculate the change in length *d* of one of the arms, using the wavelength of light used λ ; *n* then indicates in how many circles the shape has shifted:

$$d = n \frac{\lambda}{2}$$

Modelling of interference patterns

The interference shapes that occur can also be relatively easily modelled by a computer. A simplified scheme from Figure 5 is very suitable for this. For our purposes, we can omit all mirrors from it, including the semi-transparent one, and keep only the detector and two images of the source. As was already mentioned, the interference patterns are the result of light interference from these two images. Thus, for any point X on the detector, we can find the phase shift of the light beams, or in this case the difference of the path lengths from both images of the source to this point (Fig. 6).



Figure 6. The simplified scheme of the interferometer useful for modelling

We are interested in the difference of lengths of the lines x_1 and x_2 . This can be calculated by the Pythagorean theorem using the distances of the images of the sources from the interferometer d_1 and d_2 and the distances between the point X and the normal lines to the detector drawn from the images of the source $-a_1 = a_2$.

$$x_1 - x_2 = \sqrt{d_1^2 + a_1^2} - \sqrt{d_2^2 + a_2^2}$$

If we multiple the difference of lengths and the wavenumber k of the light used with the wavelength λ , we obtain the phase shift of the light beams φ .

$$k = \frac{2\pi}{\lambda}$$
$$\varphi = k(x_1 - x_2)$$

The intensity of the light *i* at a given point can be mathematically expressed from the phase shift as follows:

$$i = \left| \cos\left(\frac{\varphi}{2}\right) \right|$$

The resulting intensity will reach values from 0 to 1, which is suitable for using a colour gradient when creating a simulation. (Sources [9] and [10])

The simulation programmed using these equations is surprisingly good compared to reality. With the varying wavelength of light and the position of the images of the source, all the mentioned types of patterns can be obtained (Fig. 7). The simulation is available in sources [12] in the form of an application.



Figure 7. Interference patterns from the simulation created using the Unity Engine (green dots illustrate the horizontal placement of the images of the source above the detector; in the first and second patterns the images of the source are at different distances from the source, and in the third pattern they are at the same distance).

Homemade LEGO Interferometers

Although the interferometer may seem like a relatively sophisticated instrument that requires precisely working components, the opposite is true (of course, if we do not require any accurate measurements). The simplest version can also be made from a LEGO kit with the addition of several necessary components (Fig. 8). A laser pointer can be used as a source, basically any mirrors can be used, and a wall or paper can simply be used as a detector. The only more difficult component is the semi-transparent mirror, which can be easily purchased online; prices on the AliExpress online store are around 100 to 200 Czech Crowns. It is recommended to add a divergent lens to the interferometer, ideally just behind the laser (missing in Figure 8). The conical shape of the beams makes it easier to obtain relatively nice interference patterns. Additional diverging lens can be placed on the detector arm to enlarge the resulting image. In an emergency case, old glasses or a magnifying glass can be used as a lens (see Fig. 8) – in this case, however, the image will be magnified only after the double distance of focal length of the lens from the magnifying glass, and the image will be inverted.



Figure 8. The completed LEGO interferometer

Plans for the production of a simple mirror holder (Fig. 9) used in the mentioned interferometer are given in the sources [12] in the format for Briklink Studio 2.0 application, in which images of this module were also created (Figs. 9 and 10). The holders of the other components of the interferometer (laser, semi-transparent mirror, lenses and possibly detector) are not listed in the plans. Their appearance can vary considerably depending on the form of the components themselves; for none of them is essential to be movable, and their construction is therefore relatively simple. The required LEGO parts can be easily found according to their serial number (according to which they are referred to below) and purchased, for example, on the BrickLink website. The sources also mention the LEGO interferometer project [11], which was partly the inspiration for this project. It achieves relatively better results, but its construction is considerably more difficult, more expensive, and requires a significantly larger number of parts of the kit.



Figure 9. Mirror holder (any plane mirror approximately larger than 3 cm can be placed into the yellow part and can be fixed in place by moving the cranked yellow parts).

When creating an interferometer, it is advisable to pay attention to both its controllability and overall stability. The whole interferometer is very prone to vibrations and the interference patterns can be lost even due to completely imperceptible shocks. If the interferometer is not strong and stable enough, all vibrations in the room will significantly affecting it. It can even be affected by something as small as breath, sound, or, in the case of placement on the higher floors of the building, the vibration of the building itself. This effect can be partially avoided by a sufficiently heavy and strong interferometer base. For LEGO interferometer purposes, large LEGO base plates (e.g. part 4186a) can be used, which allow easy placement of components and can be additionally placed on a heavy table and loaded with weights to increase stability.

Equally important is the mentioned controllability of the interferometer itself, or rather the ability to focus individual beams. It is advisable to be able to rotate both mirrors and focus the beams both horizontally and vertically. Gear-wheels available in LEGO kits are directly convenient for this purpose. Their only disadvantage is usually a barely noticeable free motion, which can cause a noticeable "bouncing" of the beam on the detector when aiming it, especially in the horizontal direction. It is therefore advisable to use the smallest possible number of gears and thus, in order to achieve maximum accuracy, to use wheels with the largest possible size difference in the gears. The LEGO turntable part (composed of parts 48168 and 48452) together with the helix part (part 4716), whose rotation ratio is 56:1, is quite ideal for this purpose. The problem is the already mentioned free motion, which is really noticeable in these two parts. In the presented interferometer, the mentioned gear is used for horizontal aiming of the beams, but due to the free motion between the wheels, it is almost impossible to subsequently aim the beam vertically because even with a small movement it horizontally dejects. The solution to this problem is to use rubber bands to prevent slight unwanted movements of the final gear-wheel (Fig. 10) and thus everything attached to it, or to strictly use a "more robust" type of gears that have significantly less free motion (e.g. parts 32498, 32269, 32270). In the case of vertical aiming, this problem is solved by gravity, or more precisely, the mirror has usually sufficient weight to fix the final moving parts to which it is attached, and thus to prevent their unwanted movement.



Figure 10. The rubber band mounting parts are highlighted in red; the pull of the rubber band locks the turntable on the teeth of the helix and thus removes the free motion in the gear.

Although as a scientific tool LEGO is not completely perfect, in a calm environment it can be used to achieve acceptable results in the form of several interference patterns. It is relatively easy to obtain parallel and hyperbolic bars. Concentric circles are already significantly more difficult to create in this design, but not completely impossible. However, they require a significantly more stable and sophisticated system compared to the one that can achieve previous patterns, which due to its complexity and cost may be outperformed by other alternatives, such as the subsequently mentioned 3D printed one. Nevertheless, LEGO remains a good alternative for people without access to better, but significantly more time-consuming methods. Last but not least, the LEGO interferometer is a visual model of more sophisticated instruments and suitable for demonstration at school.

More Precise Equipment with the Help of 3D Printing Technology

The rise of 3D printing technology enables relatively cheap production of high-precision components. Thanks to the high precision of the parts, the final product is much closer to professional optical devices than the LEGO variant. It enables good and precise aiming of the beams and, thanks to the possibility of attachment to a hard solid base, it resists vibrations perfectly. The disadvantage is the long time required to produce and install all the necessary parts.

This part of the paper will try to solve at least one of these problems by providing models for interferometer production and instructions on how to use these plans effectively and make the device.

The construction described below was made of the PLA material with a 0.3 mm diameter nozzle. In Appendix [12] there are 6 prepared files for the construction of the interferometer, inspired by the design of the S'Cool LAB section from CERN [13]. This particular model consists of a total of four modules, the correct construction of which will be described gradually.

Beam splitter module construction

This is both the central module of the entire "kit" and the simplest module for production, due to the absence of moving parts. To make it, you must first 3D print the *Beamsplitter_pillar.stl* (Fig. 11 left) and *Base.stl* (Fig. 11 right) files. It is necessary to print both vertically, i.e. the position is the same as the position in Fig. 12.



Figure 11. The beam splitter holder on the left, modular base on the right

Place a semi-transparent mirror measuring 30×30 mm on the beam splitter holder (Fig. 11), which we very gently tighten with an M4 screw.

Figure 11 on the right shows the part of the base that is the same for all modules. The starshaped shape in the inner part allows the modules to be inserted in 45° steps and then to fix their position by tightening the screw by hand. Four holes are a redundant way to attach the module more firmly to the base – preferably with M4 screws arranged in a square shape with a 4 cm long edge. The final module is shown in Figure 12.



Figure 12. Completed beam splitter module

Module with a mirror

When it comes to construction, the mirror module is more complicated. You need to print twice *Knob.stl* (Fig. 13), *Pillar.stl* (Fig. 14 left), *Mirror_holder.stl* (Fig. 15 left) and *Base.stl* again.



Figure 13. Manual adjustment screw

In combination with M5 components, a fine tilt control screw is easily created.



Figure 14. The completed pillar on the left, the heat sink of the nuts with a soldering gun on the right.

Figure 14 shows the main pillar with the "L" structure for mounting the mirror. The M5 nuts and the M8 cap nut are thermally embedded in the plastic (see Fig. 14 on the right). The large nut acts as a pivot point, creating lever arms whose distance is determined by the screw (Fig. 13). This creates a system to fine-tune the mirror angle with a theoretical accuracy of 0.6 °/one turn.



Figure 15. Mirror holder on the left, assembled upper part on the right

The part of the mirror holder (Fig. 15 on the left) is adapted for mounting a molybdenum mirror with a diameter of 25 mm and a thickness of 3 mm. Subsequently, this unit is mounted on a pillar and held in place by the pressing force of rubber bands (Fig. 15 on the right) or springs.



Figure 16. Completely assembled module with a mirror

It is important to note that, for the working interferometer, two of these modules are needed.

Assembly

The last module is designed to hold a laser source with a diameter of 12 mm. Its construction is very similar to that of the construction of modules with mirrors. You just need to print *Laser_holder.stl*.

Now the modules are assembled into the final configuration of the interferometer (Fig. 17 on the left) and only the targeting process remains to put it into service.



Figure 17. A functional demonstration apparatus on the left, and the resulting interference pattern on the right.

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