How to weigh using ammeter and voltmeter or Make your own (almost) Kibble balance

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Abstract

A kilogram was redefined in 2019. For its practical implementation, more precisely for accurate mass measurements a so-called Kibble balance is used. This paper describes how to make a cheap and simple equipment that allows to demonstrate and basic principle of the Kibble balance.

Motivation: how the kilogram is measured nowadays

Effective since 20 May 2019, the SI system redefines the base units. The kilogram is newly defined using the Planck constant. The new definition is not going to be presented or commented on here. (See e.g. [1] for the exact definition, the wording with commentaries can be also found in Wikipedia [2], [3].) We will focus on the implementation of the kilogram, i.e. measuring it, or better yet weighing it.

The older definition of the kilogram was illustrative and understandable even at the elementary school level. Let's imagine what a layman could remember – it might be something amongst these lines: "Simply put the kilogram is the thing that is stored in Sèvres near Paris".

A layman probably does not imagine anything based on the new definition, possibly his only take from different interpretations might be: "The kilogram is still the kilogram I know, except now these scientists make it awfully complicated via some constants."

Perhaps this is the very maximum of what can be achieved in layman's terms. ☺ On the other hand, we probably shouldn't give up on giving our students at least some idea how the kilogram could be realised using the new definition. Meaning that at least us, physics teachers should have some clarity on this.

Several considered options how the kilogram could be realised exist, see for example a paper by M. Rotter [4]. Most reported is the Kibble balance that determines the force based on measurements of voltage and current. These can be astonishingly precise nowadays. For example, papers [5] and [6] state that voltage and current can be measured using quantum effects, specifically a so-called Josephson effect and quantum Hall effect. These allow for creation of voltage and resistance etalons with uncertainties in the order of 10^{-9}. Moreover, a very accurate value of the current can be derived from voltage and resistance. In the relations describing values of such etalons the Planck constant h and the elementary charge e occur. In the product $U{\cdot}I$ only h remains, which is the constant used in the definition of the kilogram.

These quantum physics matters won't be further discussed here, after all those are several levels above the secondary school physics. The important thing is that if we are able to precisely measure mass using voltage and current, then this measurement is linked to the Planck constant – and thus to the new definition of the kilogram.

The principle of the given measurement was suggested by Bryan Kibble in 1975, whose name is nowadays used to refer to these scales. (A formerly used name "watt balance" can be encountered as well.) The aim of this contribution is to show, using simple materials, how this device works and how is it used to measure.

Kibble balance measures a force

Right from the beginning, let us emphasize that the Kibble balance measures *a force* – a gravitational force mg when measuring weights. So, to determine the mass m the knowledge of Earth's gravity is required; it is determined using very precise gravimeters in professional measurements.

The weight of the weights is balanced by a force exerted on a conductor with electric current in a magnetic field. The situation for one current loop is shown in fig. 1.

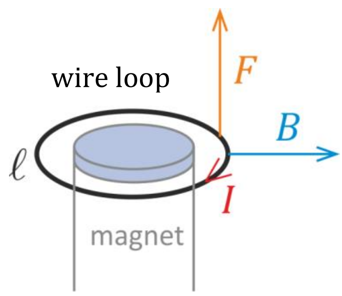

Fig. 1. The force exerted on a wire loop in a magnetic field

Provided that the magnetic induction $\vec{B}$ is perpendicular to the current and its magnitude is the same in every part of the loop, a magnitude of the force is given by a known formula $F = B\ I\ \ell$, where I is current, and ℓ is the length of the loop. In case that the magnetic induction at each point of the loop forms an angle α with the plane given by the loop, then cos α is also included in the force formula, see fig. 2.

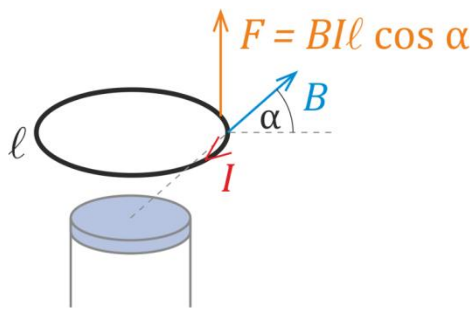

Fig. 2. The force exerted on a wire loop in case the magnetic induction doesn't coincide with the loop's plane.

The force can be determined even in a more general situation, yet such calculations rather belong to university physics.

The essential point is that while we can measure the current very accurately (at least physicists can do it very accurately using the quantum phenomena mentioned above), the measurement of the length of the loop and the magnitude and direction of the magnetic induction is not nearly as accurate. (In reality, it is not a single loop but rather a coil that is used for the measurement, and for an accurate measurement we would need to know the geometry of the whole arrangement and distribution of the magnetic field in great detail.) So, we wouldn't even be able to measure the force accurately enough - if it weren't for Kibble's truly brilliant idea.

Kibble’s idea

The basis of B. Kibble’s idea was keeping the same magnetic field and moving the loop (resp. a coil) up and down, as shown in fig. 3, while measuring the induced voltage.

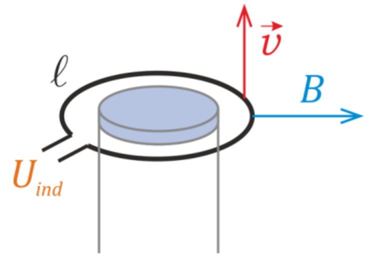

Fig. 3. Induced voltage is measured while moving the loop (through the same magnetic field).

The magnitude of the induced voltage is $U = v\,B\,\ell$. Let us note that in secondary school physics this formula is derived for a straight conductor moving through a homogenous magnetic field. In the case of our loop the loop can be divided into small parts, consider each of them to be sufficiently straight, voltage on each part is $\Delta U = v\,B\,\Delta\,\ell$ and finally sum all these parts back together, therefore the voltage is indeed $v\,B\,\ell$.

This formula can be alternatively derived from the law of electromagnetic induction: During a short time Δt, the loop moves by a distance of $v\,\Delta t$ in the vertical direction, see fig. 4. Through the lateral surface of the thought cylinder with bases formed by the loop in times t and $t+\Delta t$, flows a magnetic flux $\Delta\Psi = B\,\Delta S = B\,v\,\Delta t\,\ell$. The magnetic flux passing through the loop decreases by $\Delta\Psi$. Then just calculate $\Delta\Psi/\Delta t$.

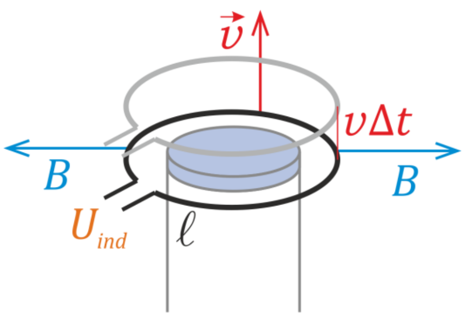

Fig. 4. On derivation of induced voltage U_{ind} from the law of electromagnetic induction.

If the magnetic induction is not parallel to the plane of the loop, only a component $B\cos\alpha$ applies in the formula for the induced voltage, see fig. 5.

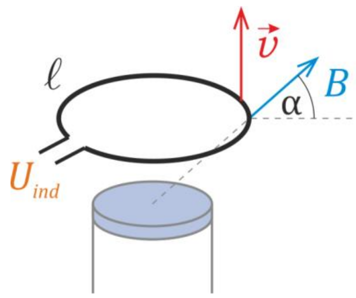

Fig. 5. Voltage induced on the loop in more general position is $U = v\,B\,\ell\cos\alpha$.

What makes Kibble's idea so great

The force exerted on the loop in the situation described in fig. 2 is

$$F = I\,B\,\ell\,\cos\alpha\ . \qquad (1)$$

Voltage induced on the moving loop in the situation described in fig. 5 is

$$U = v\,B\,\ell\,\cos\alpha. \qquad (2)$$

Dividing formula (1) by formula (2) yields

$$\frac{F}{U} = \frac{I}{v} \quad \Rightarrow \quad F = \frac{U\,I}{v}\ . \qquad (3)$$

That means that to determine the force no magnitude of magnetic induction, nor length of the loop, nor the direction the magnetic induction forms with the loop's plane is required!

Simply put – firstly move the loop at a known speed v, and measure the induced voltage U. Then suspend the measured weight from the loop and set the current I so the force F that the magnetic field exerts on the loop balances the object's weight. Formula (3) then gives the weight. The whole construction is usually realized as scales with a balance as known from elder lab scales. Of course, a coil with many turns is usually used in loop's stead.

The whole derivation can be made under more general presumptions, still it turns out that the result is independent of the geometric layout. For instance, the magnet's pole doesn't have to lie on the axis of the coil, so the magnitude of magnetic induction is different in different parts of the loop – even in this scenario, formula (3) holds.

Let us note that formula (3) can be simplified to $F\,v = U\,I$. Formulas on both sides are measured in watts – that is apparently the reason why the device used to be called "a watt balance". This denotation is actually quite imprecise, because both the force and the speed are measured in different stages of the experiment, very similarly to the current and the voltage. Let us add that the change in name to the Kibble balance was decided by a committee of the International Bureau of Weights and Measures in 2016, two months after the death of B. Kibble.

Demonstrating the principle of a Kibble balance: professional results and our ambition

A professional Kibble balance (for example see figures and photos in the paper [4] or in Wikipedia [7]) are accurate up to the order of 10^{-8} nowadays and are of course tremendously expensive. Physicists from the National Institute of Standards and Technology (NIST) and from the University of Maryland have already built a demonstrational version of Kibble balance six years ago. They describe it as a construction out of LEGO in the paper [6], nonetheless it implements much more sophisticated technology as well. They are able to measure the coil's position with a precision up to 50 µm using a "shadow sensor"; their coil consists of three thousand turns. The balance's movement is stimulated by a current through a coil on the other arm, once again in a permanent magnet's field. Their overall precision was better than approximately 1 %.

What ambitions could a Czech physics teacher have?

- To create a cheap equipment "from laths and available materials"
- Concerning precision: "I wish it works out at least in the right order…" (Let's say 20 to 30 %.)

„Kibble balance of a Czech physics teacher" – first attempts

First theoretical estimates obtained from relations (1) and (2) showed that by using neodymium magnets and a coil of several hundred turns, the principle of Kibble balance should be demonstrable. Also, from the first experiments, when the magnet in the coil was moved by hand, an order of magnitude agreement of the F/U and I/v ratios in relation (3) was obtained.

The first design of the " Kibble's scales of the Czech physics teacher" actually had a balance made of a wooden lath. The coil was wound on a sawed-off plastic syringe and had 200 turns wound rather wildly over each other. (In amateur radio practice, such coils were actually called "wound wild".) The coil was attached to the lath by inserting the tip of the plastic syringe into a hole drilled in the lath. The balance was counterbalanced by a nut tied to it with a bell wire. A neodymium magnet, 1 cm in diameter and 2 cm long, was held by its own magnetic force to a steel plate bolted to the base plate of the balance. The weight was simply placed on the other arm of the balance. The whole design is shown in fig. 6.

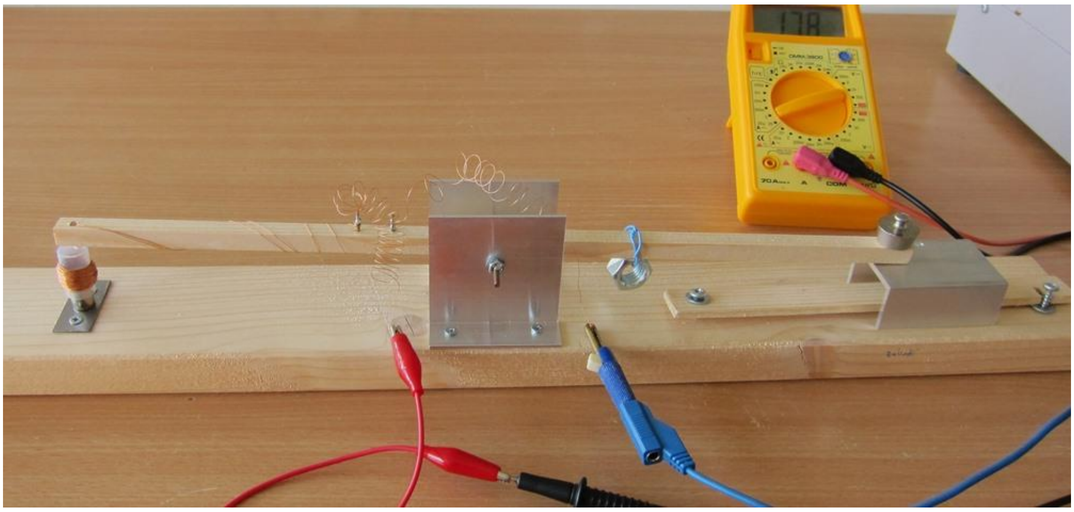

Fig. 6. The first design of the demonstrational equipment.

When balancing the weight, the coil was powered by a laboratory power supply and the current was measured with a multimeter. For a 20 g weight, the balance was balanced at a current of 180 mA. The force $F = mg$ was thus approximately 0.196 N, and the ratio $F/I \approx 1.09$.

The movement of the coil in the second part of the experiment was realized in this design by a rotating "cam" made of bent wire, rotated by hand using a crank, see fig. 7.

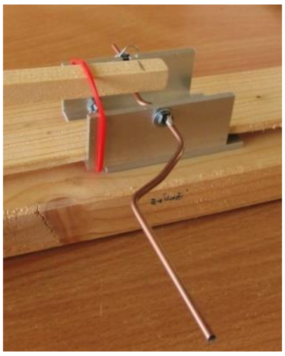

Fig. 7. The „cam“ made from a wire oscillates the balance up and down, when rotated.

The voltage induced in the coil was detected by LabQuest2 with a 6 V sensor, and the values were subsequently exported to Excel. It was possible to determine the period of oscillation T and the amplitude of the voltage U from the voltage waveform. Assuming that the oscillation waveform is harmonic (i.e. "sinusoidal"), the maximum speed of the coil can be determined from the amplitude A of the coil oscillations and the angular frequency ω = $2\pi/T$ as $\upsilon = A\,\omega$. The amplitude of the oscillations is determined by the "eccentricity” of the cam; it was about 0.5 cm in our case.

The period of oscillations in this particular case was determined from the recorded voltage waveform to be about 0.32 s, and the calculated maximum speed was about 0.1 m/s.

The recording of voltage versus time shows that the voltage waveform is unfortunately not exactly harmonic, see fig. 8.

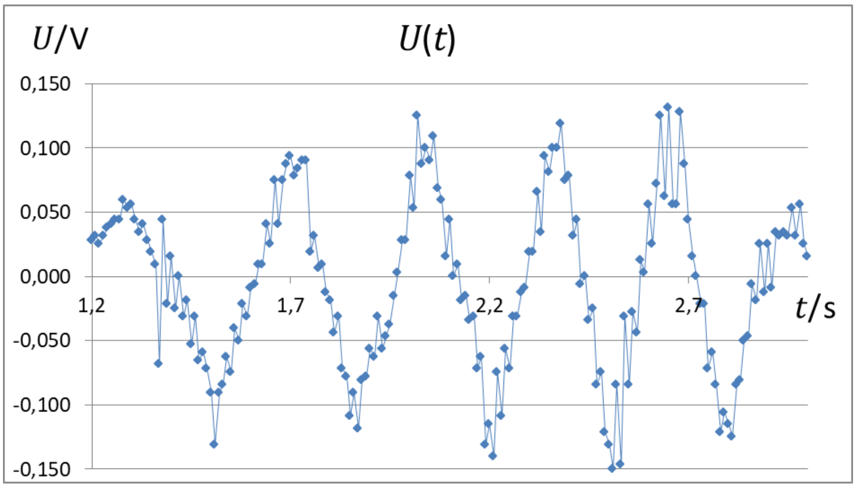

Fig. 8. The voltage waveform in the first version of the balance.

Obviously, too much cannot be expected from this version of the device. However, if we estimate the maximum voltage values to be about 0.14 V, we get the ratio $U/\upsilon \approx 1.4$.

It follows from formula (3) that theoretically $\frac{F}{I} = \frac{U}{v}$. In our case the ratios are 1.09 and 1.4, so they differ by about 30%. In fact, this simple version of the device already fulfils the ambitions set out above. However, particularly the record of the voltage waveform in fig. 8 indicates that the design should be improved.

When some reflection is done, the problematic parts of the first version of our tool seem rather obvious:

- In a coil wound "wild", the induced voltage significantly depends on the position of the magnet relative to the coil, in addition to the speed of motion. (Thus, the voltage may not be maximal when the speed is maximal.)
- When manually turning the crank, the speed of rotation is not uniform, hence the oscillations of the balance are apparently non-harmonic. (In addition, the uneven surface of the lath where the "cam" slides over it may contribute to the " serrated" voltage waveform in fig. 8.)
- Evidently, the estimation of the amplitude of the oscillations is rather inaccurate. At an amplitude of 0.5 cm (moreover, measured with an ordinary ruler, being a little bit of a "guesstimate"), a deviation of 1 mm means a relative error of 20%.

The second version of the Kibble balance model attempted to mitigate these sources of error.

The second version of the balance

In the second (and the last so far) version of the Kibble balance model there is:

- An improved coil. Once again, it is wound on a piece of plastic syringe, but carefully this time, turn by turn (with enamelled wire of 0.224 mm diameter), in two layers, 330 turns in total. The length of the coil is 5 cm. In the equilibrium position, the pole of the magnet is in the middle of the coil. (The magnet is neodymium, 1 cm in diameter, 4 cm long, composed of two 2-cm-long magnets.)
- In the phase of the experiment, when the coil is moving, the pendulum oscillates due to a spring. (A spring 15 cm long, 1.5 cm in diameter, from an older school set, was used. The stiffness of the used spring was about 50 N/m, the exact value is irrelevant.)
- The amplitude of the oscillations was determined using a laser mounted on the arm of the balance, see Figure 9. The laser beam was incident on a distance scale, and in this particular measurement the scale was 280 cm away. The maximum displacement of the trace could therefore already be determined by observation with the naked eye (it is useful to have assistants for this); for the actual measurement (which was made by the author alone), the movement of the trace was filmed with a high-speed video camera (210 fps), the amplitude was then measured in the software Tracker.
- In addition, the measured weight was not simply placed on the balance but was suspended on a thread. This improved the accuracy of the distance of the weight from the rotational axis.

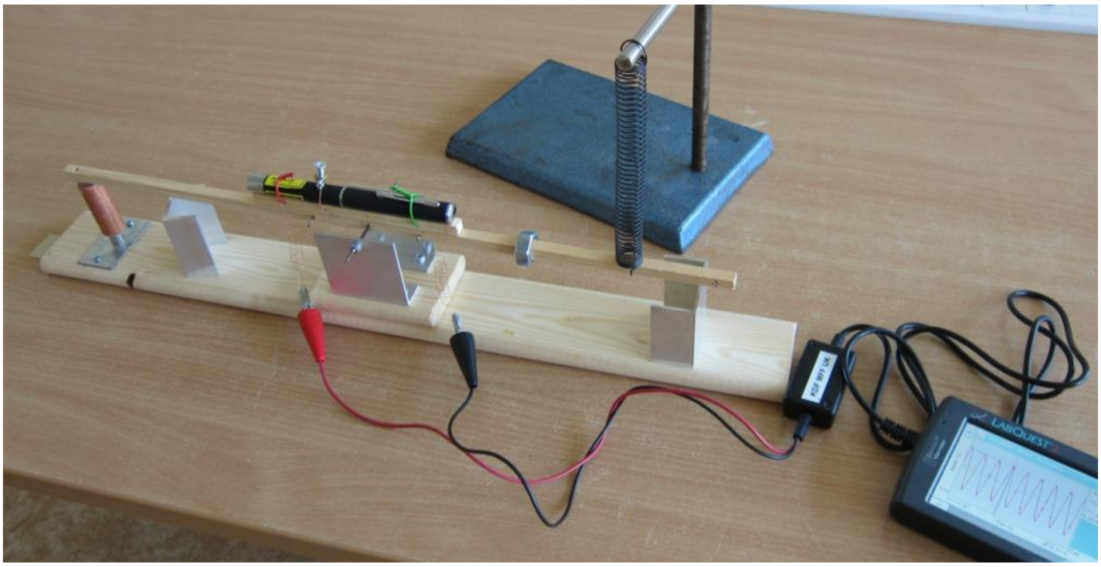

Fig. 9. Execution of the second version of the Kibble balance model - the phase of measurement with coil movement.

Measurement using the second version of the balance

In the force measurement, a 10 g weight was balanced at a current of 163 mA. The unbalance was clearly visible at a current change of 1 to 2 mA. The force/current ratio is $F/I \approx 0.602$ N/A.

When measuring the voltage induced on the moving coil, the period of the oscillations was about 0.20 s. (The frequency of the oscillations determined from the frequency spectrum was $f \doteq 5.005$ Hz. The fact that it worked out this way is coincidental, of course; the frequency depends on the distance from the axis at which the spring is attached to the balance.) The oscillations have relatively small dampening: in 20 s their amplitude dropped to about one half. As Fig. 10 shows, the oscillations can already be considered as harmonic " with reasonable accuracy ", at least when the graph is assessed subjectively.

One note: In the graph the measured values are shown in blue, the red curve underlying them is the fitted harmonic function, more precisely a sine wave with exponentially decreasing amplitude. The parameters of the fitted curve can be set "manually" or using the Solver in Excel. This way we can obtain a more accurate value of the amplitude, of course we can read it directly from the graph for an approximate estimation.

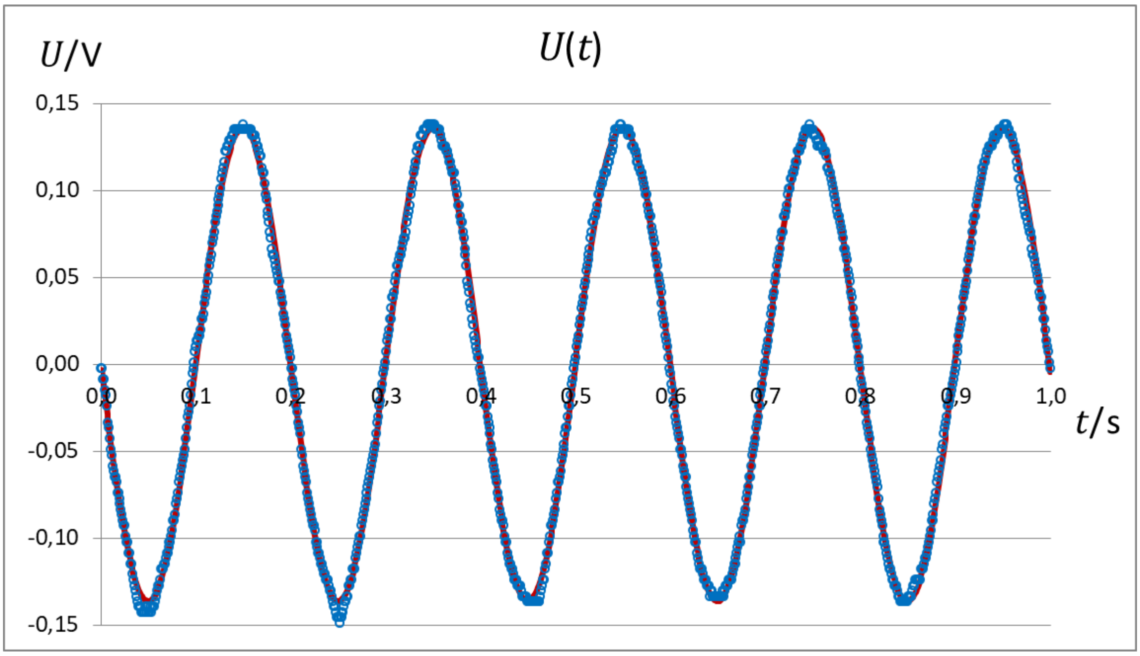

Fig. 10. The voltage induced on the moving coil.

In this particular measurement, the amplitude of the coil oscillations was 7.0 mm, resulting in a maximum velocity of about 0.22 m/s. The voltage amplitude determined by the Solver was 0.136 V; from the graph we would probably read it as 0.14 V. The ratio of voltage to velocity is therefore $U/v \doteq$ 0.618 V/(m/s).

By comparison, we see that U/v and F/I differ by only about 2,7 %. (Were we to use a less accurate voltage amplitude value read directly from the graph, the deviation would be less than 6%.)

We can also calculate U/v values from other regions of the $U(t)$ graph. For instance, in a specific measurement, the U/v values detected at around 5 s differed from the F/I by about 3.9%, and for values detected at around 12 s the deviation was about 1.7%.

We can preliminarily conclude from these measurements that with our model of the Kibble balance **we achieve an accuracy of about 2 to 4%**, generally in the order of units of percent. Of course, this is not sufficient for actual accurate weighing, but the design can be used to demonstrate the principle of realizing the kilogram using the Kibble balance.

Other potential improvements?

The accuracy of our model of the Kibble balance could perhaps be further improved. (After all, the fact that the ratio of U/v to F/I has always been slightly greater than 1 in previous measurements suggests that there is some yet unknown systematic error.)

One source of error may be the measurement with the Differential Voltmeter (DVP-BTA) sensor coupled to the LabQuest. The voltages with an amplitude of 0.1 to 0.15 V that are induced on the coil are rather small for this sensor. (The manual states that the sensor has a sensitivity of about 3 mV and a noise of up to 18 mV.) If we do not want to wind the coil with a much larger number of turns, we could build a voltage amplifier with a gain of, say, 10 (up to 40 would be possible, since the sensor's range goes up to 6 V), using an operational amplifier, this would be a relatively simple design.

It would be a good idea to verify how accurately the multimeter measures the current when refining the measurement. (The manual for the used type of multimeter states an accuracy of ±1.2% for current measurement.)

Attention also needs to be paid to measuring the amplitude of the displacement, this may be critical for accuracy. For further refinement, it would probably be necessary to no longer have the coil rigidly connected to the arm, but to suspend it on some sort of a hinge... but that's where the design would become complicated.

Conclusion: What it is and can be good for

Several answers to the question “what is it good for” can be given.

Firstly, the described device may perhaps bring us closer to how a kilogram is "weighed" these days and the Kibble balance will no longer be something completely strange and mysterious to us.

Secondly, it can be a topic for pupils' and students' projects. Such a project can be carried out both in a simple version, where an accuracy of an order of thirty percent or more would be enough, and in more demanding versions up to very sophisticated constructions, greatly exceeding the model of the balance described in this paper.

And finally, there is a lot to be learned from making, refining, and measuring with this device. From my own experience I can mention for example:

- Using digital technology for data collection.
- Using software and applications for measurement data analysis (e.g. the usage of Solver in Excel).
- Training of patience while winding the coils. (☺)
- Training of creativity while reflecting on how to improve accuracy.
- And finally, a training and development of humility that after intervening with the design the resulting accuracy not only did not improve, but has rather deteriorated.

For those of you who found this post an inspiration to build, either by yourself or with your students, a similar model that allows you to weigh with an ammeter and voltmeter, I wish you to enjoy it as much as possible.

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