More ideas from Malá Hraštice (easy experiments with famous names in the background)

LEOŠ DVOŘÁK
PETR KÁCOVSKÝ
Department of Physics Education, MFF UK, Praha

Abstract

This contribution describes a few experiments which are connected to the names of famous physicists: 1) “almost” Rutherford experiment, in which repulsion of macroscopic objects is demonstrated, 2) easy verification of Ampére law of total electrical current, 3) repulsion of magnets where poles of long thin magnets repel each other analogously to Coulomb’s law; however, the situation with shorter magnets is more difficult, and 4) the Oersted’s experiment. All these experiments are simple and should mainly help to grasp the corresponding concepts and laws.

Introduction

The spring gatherings for future physics teachers, held annually for the last 22 years by the Faculty of Mathematics and Physics of the Charles University, are great opportunities for trying out less traditional ideas for experiments which can be used in daily teaching of physics. We will not describe the gathering itself in detail because there are other contributions from the past in which you can find more information, for example [1]. Ideas for experiments, which came up from different years of gatherings, were described in a number of contributions at the Physics Teachers´ Inventions Fair – contributions can be found under the author’s name in the “supercollection” [2] on the web.

In year 2018 the main theme for the gathering was “Story, or the way of discoveries and inventions”. Therefore, the experiments were in some way connected to the names of discoverers and inventors, after which the laws of physics or known experiments are named. Image of the gathering´s atmosphere, some of the experiments and constructions made by participants can be found on the website [3]. In this paper we will describe experiments and tools, used and tested by us at this year´s Hraštice; some of them were improved and extended later. Experiments or groups of experiments 1,2 and 4 were designed and tried out first by the first author (L.D.), experiments in group 3 are products designed by the second author (P.K.), who in addition led the whole scientific program.

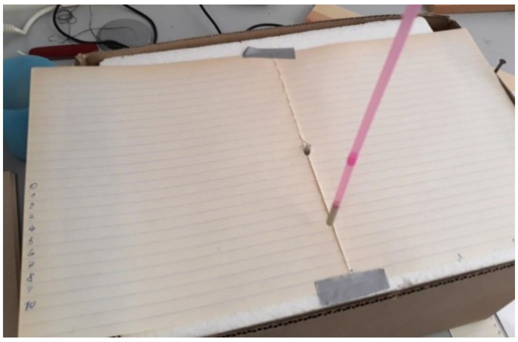
1. The “almost“ Rutherford´s experiment

In schools Rutherford experiment with scattering of alpha particles on gold atoms nuclei can be described theoretically or illustrated on some pre-programed models. Would it be possible to make a macroscopic model, which would illustrate this experiment?

Sometimes we simply use a model with a ball rolling on the table, when the ball approaches a “hill” on the table, the direction of the ball changes. Could we demonstrate the repulsion in “contactless” way, like in a real repulsion of alpha particles on gold nuclei? The best analogy would be an electrostatic repulsion, but this method would require really light balls and the charge would apparently "run away" – for the experiment in the "field conditions" this method did not seem feasible. It became clear that better option would be a larger repulsion force. This led to the idea to use a repulsion of magnets, or more precisely poles of long magnets. In this case we have an analogy of the Coulomb´s law (see [4]).

The experiment described below, tested at Hraštice, is more likely an illustration for now and its realisation will need further improvement. However, it can offer a qualitative description of scattering.

Instead of gold atom nucleus we will use a long bar magnet (composed from more neodymium magnets, in our case 5 mm wide and 2,5 cm long). The magnet was positioned perpendicular to the surface, its upper end was standing out a little from a polystyrene plane which was layed on a paper box, see pic. 1.




Picture 1: Model of Rutherford´s experiment, in which the repulsion acts between the poles of the magnets

Second magnet is analogous to a moving alpha particle. It is hung on a long guidance like a pendulum. In the beginning we displace the magnet to the side (to the left side from fixed magnet in the picture) and release it so it moves parallel to the lines on the paper placed on the polystyrene plane. The repulsion from the fixed magnet will cause deviation from the initial direction. As in the original Rutherford experiment the deviation is bigger the closer the moving magnets "get to" the fixed one. (In terms of physics, the smaller the impact parameter the bigger the deviation.)

The experiment, designed as described, came along with a number of problems. For repulsion to act in a way that fits Coulomb´s law, moving magnet needs to be long. However, that kind of magnet is heavy, so "reasonable" deviation is achieved at small distances from the fixed magnet. Great force present at smaller distances caused the moving magnet to considerably oscillate (the guidance was made from string). Reducing the oscillations has been achieved by using smaller magnet inserted in plastic straw which was used as guidance. However, in this case the force between the magnets is not proportional to $1/r^2$. If the experiment were to be at least partly quantitative, we would need for given setup to be able to calculate or measure the force as function of distance, deduce what trajectories are predicted by given theory and compare them with results of the experiment. In addition, magnet on a guidance can be thought of as free particle only in case of infinitely long guidance and we don't have infinitely high schools so… ☺

It is clear, that this experiment is for now more of an inspiration – but it is worthy of future development and improvement.

2. Ampère's law

Little bit of theory – and how it can be easily looked upon

Ampère's law states that line integral of magnetic field intensity around closed curve *c* is equal to total current I_{tot} passing through the surface *S* which is enclosed by the curve *c*:

$$\oint_c \vec{H} \cdot d\vec{r} = I_{tot.} \tag{1}$$

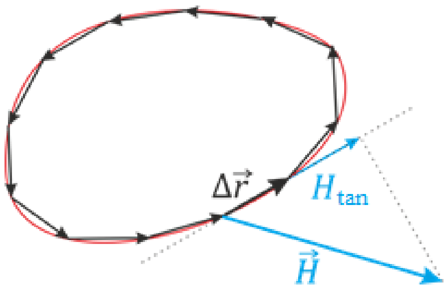
This is a formula from university level physics, for example see textbook [5]. Usually in classes this law is given more as a statement, and if there are some illustrations they are carried out with difficult experiments. In addition, in the first year of studies when this topic is discussed, line integral is a new and sometimes difficult concept to grasp for students. Could this law be verified (in some particular situation) by an easy experiment? Could we also illustrate and explain what the line integral is while doing so?

We can start from the idea that the integral is actually "sum of a large number of smaller parts". (Mathematicians will hopefully forgive us this really vague definition… Following thoughts could be more precise, but for the understanding of the measurement principle it is not needed.) We will divide the curve to smaller parts and sum up the corresponding contributions.

Of course by "summing up a large number of smaller parts" we will not get and exact value of the integral, but if the divided parts of the curve are small enough then the error of our measurement will be small enough too. Integration can be at least approximately replaced by the summation:

$$\oint_c \vec{H} \cdot \mathrm{d}\vec{r} \doteq \sum_{i=1}^{N} \vec{H}_i \cdot \Delta\vec{r}_i = \sum_{i=1}^{N} H_{\mathrm{tan.}\,i}\, \Delta s_i \tag{2}$$

where $\vec{H}_i$ is a vector of magnetic field intensity of part *i* of the curve *c* and $\Delta\vec{r}_i$ is a small vector, which approximately represents a part of the curve *c*. (See pic. 2, for simplicity we do not use subscript *i*.) $H_{\mathrm{tan.}\,i}$ is a projection of this magnetic field intensity to the tangential direction, therefore to the direction of the vector $\Delta\vec{r}_i$. Length of the vector $\Delta\vec{r}_i$ is given as $\Delta s_i = |\Delta\vec{r}_i|$.



Picture 2: Line integral is replaced by summation through illustrated parts of the curve

How are these theoretical thoughts connected to the experiment and verification of Ampère's law? In fact, very closely. They give us a manual on **how to measure line integral on the left side of (1)**:

All we need to do is divide the curve to smaller parts, on each one of them measure the value of magnetic field intensity $H_{\mathrm{tan.}\,i}$ in the tangential direction to the curve, multiply it by the length of corresponding part of the curve Δs_i and then sum up all the contributions.

In reality we measure magnetic field induction $\vec{B}$ instead of magnetic field intensity $\vec{H}$ and later we consider that $\vec{H} = \vec{B}/\mu_0$. We will determine the integral on the left side of Ampère's law from measurements as

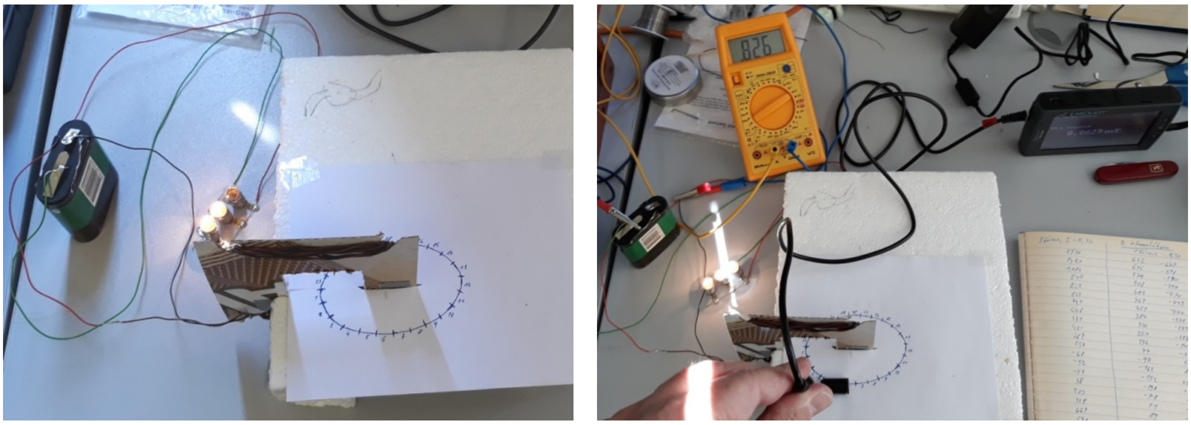
$$\oint_c \vec{H} \cdot \mathrm{d}\vec{r} \doteq \frac{1}{\mu_0} \sum_{i=1}^{N} B_{\mathrm{tan.}\,i}\, \Delta s \tag{3}$$

(Parts of the curve had same length in our case, hence Δs without subscript i.) And now, how to carry it out in the form of an experiment?

Little bit of measurement – step by step…

Easy way to measure the summation on left side of formula (3) is shown in the pic. 3. The curve drawn on the paper was divided on 25 parts of equal length, in our case one part was 1 cm long. Through the surface, enclosed by the curve, several current-carrying conductors are passing through. (In our case it was a "coil" wound up on a piece of cardboard with 14 loops.) The current was restricted by a few small lightbulbs connected in parallel configuration and was measured by a multimeter.

Magnetic field induction was measured by magnetic field sensor made by the Vernier company connected to their Labquest 2 system. The probe for measuring magnetic field induction was always turned in the tangential way to the curve – so we could measure "tangential value" od B_{tan}.

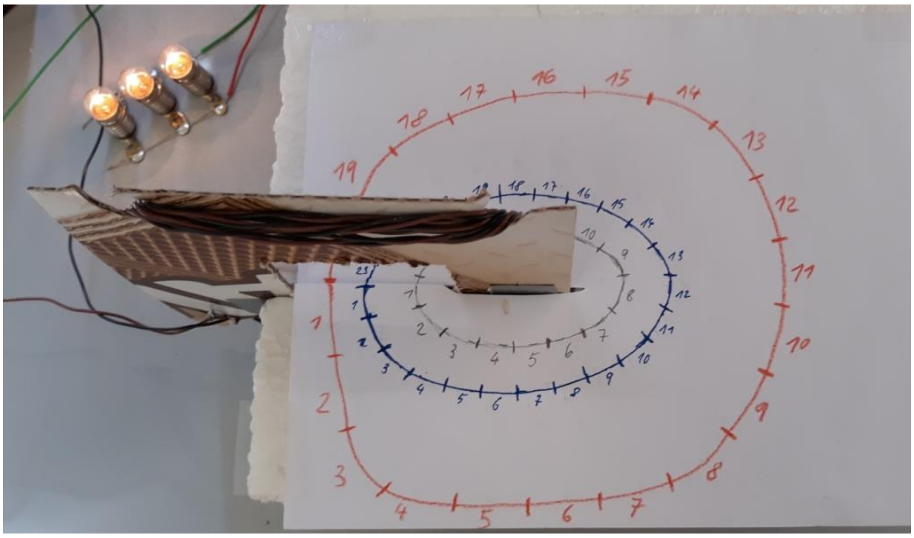


Picture 3: Experimental verification of Ampère's law

The measurement requires a little bit of patience. We needed to place the probe onto all 25 parts of the curve, write down the value of the magnetic field induction there and then sum all these values up (and also multiply them by the length Δs = 1 cm).

The measurement was done for three different curves, see pic. 4. (Outer red curve had all parts 2 cm in length.) The measurement was done twice for every curve, during the second measurement the probe was rotated by 180°.

What was the outcome?



Picture 4: The measurement was done for three different curves

Results of the measurement

The current from the battery was approximately 0.8 A, with 14 loops we get total current at approximately 11.5 A. Table 1 shows comparison between values of total current (14-times the current measured with a multimeter) and values given by the summation of the contributions from all of the curve´s parts (from the experimentally determined values of line integral $\oint_c \vec{H} \cdot \mathrm{d}\vec{r}$).

Table 1: Comparison between the values of $\oint_c \vec{H} \cdot \mathrm{d}\vec{r}$ determined by the experiment and the total current

Curve	Result of the measurement $(\frac{1}{\mu_0}\sum_{i=1}^{N} B_{\text{tan.}\,i}\,\Delta s)$	Total current ($I_{\text{tot.}} = 14I$)	Difference in %
Blue	12,1 A	11,6 A	4,3 %
Black	12,2 A	11,6 A	5,1 %
Red	11,6 A	11,0 A	5,5 %

As we can see, the values are in agreement with the accuracy approximately five percent – given that during the measurement the magnetic field induction probe was placed onto parts of the curve by hand it is quite a good result.

The whole problem discussed here obviously matches more university level and will be certainly suitable in the opening courses about electricity and magnetism for future physics teachers. This measurement will perhaps help to illustrate what a line integral $\oint_c \vec{H} \cdot \mathrm{d}\vec{r}$ is all about.

However, we should point out that previously described experiment will need future improvements. For example, what would be the results of this experiment if the curve was not planar but spatial?

3. Repulsion of the magnets

In the context of stationary magnetic field students in high school are gradually getting familiar with the formulas for calculating magnetic field induction near a straight current-carrying conductor, magnetic field induction inside of a solenoid or forces acting on the conductor in an external magnetic field. However, one of the most basic question which students can ask still lies unanswered – on what is the magnitude of the force between two magnets dependent?

Gilbert´s model

If we were concerned about the forces between point charges, Coulomb´s law would give us a clear answer. However, magnetic interaction is more difficult and it can be dependent on size, shape, material, distance or mutual position of the magnets. To make it possible to work with some numerical estimations, we will use the analogy with before mentioned electrostatic interaction and so called Gilbert´s model approximation in this experiment. This model is based on the idea of magnet as coupling of positive and negative "magnetic charge", which are identified as north and south magnetic pole [6]. Based on this idea we can find the formula for magnetic force acting between two identical cylindrical bar magnets with length l and base radius R (see [7]),

$$F(x) \doteq \frac{B^2S^2}{\pi\mu_0}\left(1+\frac{R^2}{l^2}\right)\left(\frac{1}{x^2}+\frac{1}{(x+2l)^2}-\frac{2}{(x+l)^2}\right), \tag{4}$$

where S is the area of cross section of magnets, μ_0 is permeability of the vacuum and B is magnitude of magnetic field induction´s vector in immediate vicinity of one of the magnetic poles. Magnets are aligned axially, so their poles are at a distance $x >> R$ of each other. From (4) we can see, that for $l \to \infty$, the situation can be percieved as interaction of two closest "monopoles" (with emphasis that nothing like magnetic monopole does exist!), so we get formula

$$F(x) \to \frac{1}{\pi\mu_0}\frac{B^2S^2}{x^2}, \tag{5}$$

which at least formally reminds us of Coulomb force.

Setup alignment

To verify formula (4) we can use simple measurement of repulsive force using laboratory scales. One of the magnets is positioned vertically a few centimetres above the area of the scales, second is attached to the holder which can be used to change the distance x between the ends of the magnets (pic. 5), the attachment on the magnets was easily made from plastic straws.

Picture 5: Measuring of the force between two magnets (l = 25 mm, R = 2.5 mm)

It has been proved that this experiment should be carried out by consecutively reducing the distance between the magnets, not the other way around. For this measurement, it is crucial that the magnets are in fact aligned axially, so we would avoid tilting to the side due to growing force between the magnets. When this effect begins to occur it is time to end the measurement.

How did it end up?

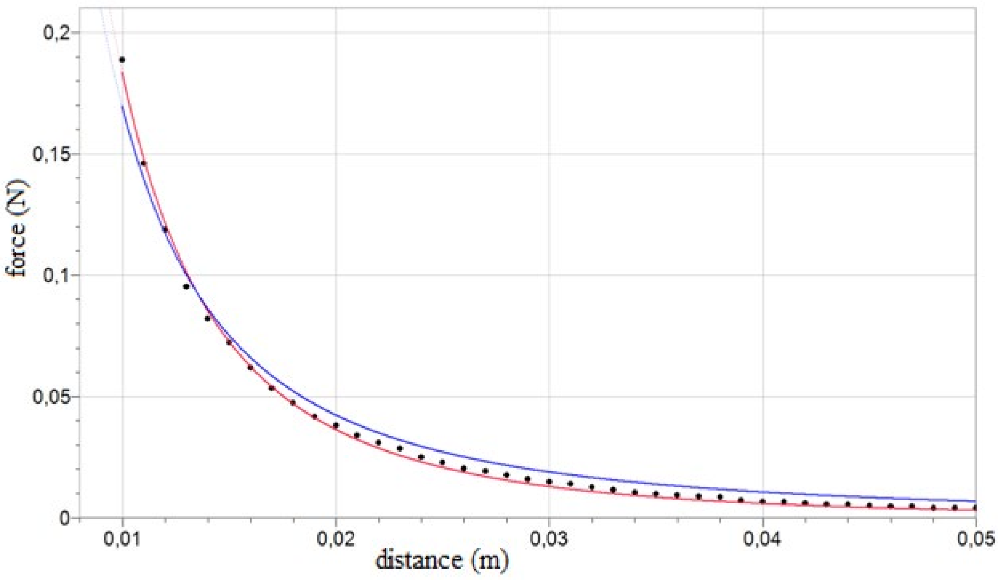
Measured data are shown in the pic. 6, measured points are black. Red curve represents fit $F = F(x)$ by using formula (4), blue curve represents fit by using formula (5). It is clear, that corrections due to finite dimensions of the magnets given by formula (4) can better describe the measured data.

By using the program Logger Pro the data was fitted by function $F(x) = A\left(\frac{1}{x^2} + \frac{1}{(x+2l)^2} - \frac{2}{(x+l)^2}\right)$ and we were able to get the numerical value of $A = 2{,}31 \cdot 10^{-5}\ \mathrm{Nm}^2$ so we could estimate the magnitude of magnetic field induction in the immediate vicinity of each of the magnetic poles:

$$B = \frac{l}{R^2}\sqrt{\frac{A\mu_0}{\pi(l^2 + R^2)}} \doteq 0{,}48\ \mathrm{T}.$$

This value is in agreement with the results stated in [8], where it is also explained why the values of remanence (stated by the magnets manufacturers) are significantly higher.

The experiments with the two flat magnets (l = 5 mm, R = 15 mm) have also shown that the assumption $x >> R$ is essential for this measurement. Measured values for a pair of flat magnets are not corresponding with the behaviour given by formula (4), because approximation by Gilbert's model is completely failing in immediate vicinity of magnets with significant pole area (we could say significantly “non-point like poles”).



Picture 6: Measured values of repulsive force as function of the distance

4. Oersted´s experiment and magnetic field induction

The Oersted´s experiment is usually demonstrated using a compass: near the conductor, the compass needle will deviate from magnetic north after turning on the current in the conductor. For significant deviation a large current is usually needed – even after that the needle might not be pointing perpendicularly to the conductor, as we would expect from textbook pictures of induction lines around the conductor under current. The magnet needle is still being affected by Earth´s natural magnetic field which can often be the dominant factor.

The challenge for us can be, how to prepare easy experiments in which:

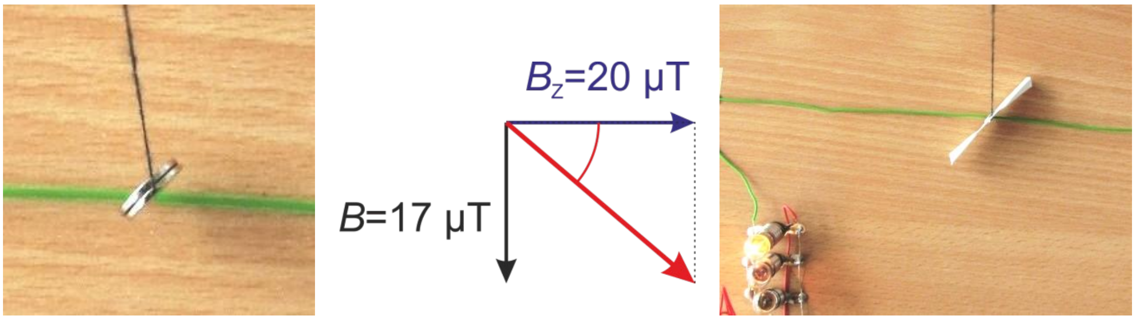
a) The deviation of compass needle or other magnet is significant even for smaller currents,

b) The compass needle shows the direction of magnetic field induction exactly perpendicular to the conductor.

If we start with the formula for magnetic field induction near a long (in theory endless) conductor.

$$B = \mu_0 \frac{I}{2\pi R} \tag{6}$$

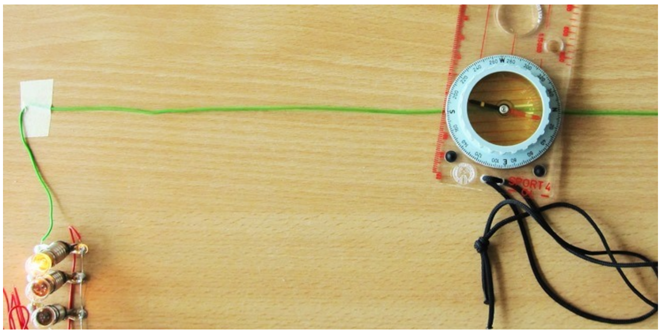
we can see that for stronger effect of the generated magnetic field induction smaller R is needed, so we need to place magnet (or compass) as close to the conductor as possible. We will use small neodymium magnets. For example, we can get two flat magnets with diameter of 1 cm attached to a thread (see experiments in [4]) as close to the conductor as R = 7 mm. Substituting into the formula (6) with current of 0.6 A we get a value of $B \doteq 17$ μT. The magnet with the axis initially parallel to the conductor will turn around for almost 45°, because the horizontal part of the Earth´s magnetic field induction is here around 20 μT. The situation is illustrated in the picture 7.



Picture 7: Turning of small magnets near the current-carrying conductor

If we want to see the direction of the magnets better, we could insert a piece of paper between the magnets representing a compass-like "paper needle", as shown on the right part of the picture. (However, be careful this paper needle is not pointing in north-south direction but west-east, it is perpendicular to the direction of the magnetic field induction.) We should point out, that this photography was taken inside of the building, where field was affected by ferromagnetic materials inside of the building, tables etc. In our case the horizontal part of the Earth´s magnetic field at the site of the measurement was less than 10 μT, therefore the deviation for about 45° happened with the current around 0.3 A.

Why the compass cannot show such a significant deviation, even if we place it immediately above the current-carrying conductor? As shown on picture 8, the middle part of the compass needle is near the conductor, but the poles of the needle are getting further away from the conductor due to deviation, therefore the poles are located in weaker magnetic field. That is why the compass needle cannot show such deviation as small neodymium magnets.

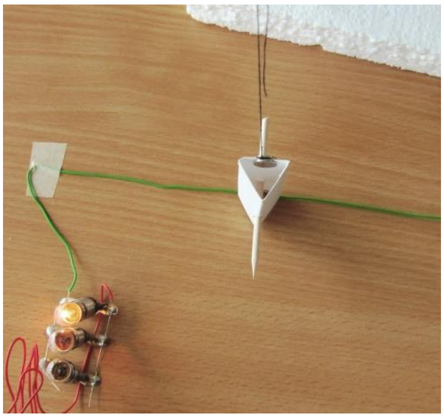
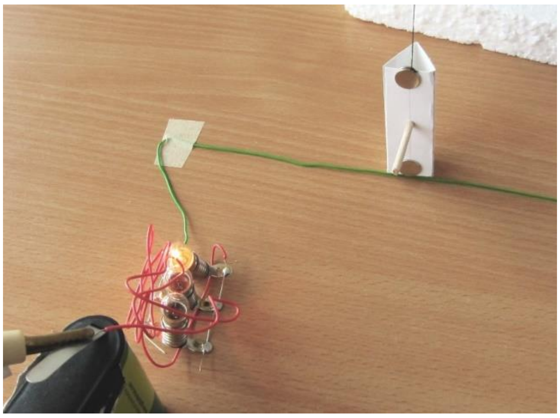


Picture 8: Even if we place the compass immediately above the current-carrying conductor, the poles of the needle are getting significantly further away from the conductor

And how can we show, that magnetic field is in fact perpendicular to the conductor? We can either use a really large current or somehow remove the effect of Earth's magnetic field – or we can try to compensate for it in some way. Following this idea, as shown on the picture 9, has proven to be surprisingly working.

Two pairs of identical flat magnets are attached to a folded piece of paper hung on a thread, one pair is pointing in the opposite direction. That is why Earth´s magnetic field (which can be considered homogeneous here) is not turning the folded piece of paper itself.

Of course, the lower pair of magnets is closer to the conductor. This pair of magnets is significantly more affected by the generated magnetic field of the conductor, than the upper pair – so it will turn our paper construction. Little piece of skewer or toothpick is pierced through the paper construction and is showing that magnetic field induction is perpendicular to the conductor.

Picture 9: Paper construction showing the direction of the magnetic field induction near the conductor. Direction of the current on the left and right pictures is opposite.

Conclusion

All previously shown experiments will get further improvements and we intent to spend more time on them. We will be thankful for your comments, on technical realisation, on involved physics or their use during teaching.

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Further efforts on development of physics experiments, which will help in removing the barriers in understanding of physics concepts and laws, including this this paper, were also supported by program University research centre UK č. UNCE/HUM/024.

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