Magnets and inductors with current

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Electromagnetic pendulum

Let us focus on a single device – the electromagnetic pendulum (figure 1, figure 2) whose weight – a magnet is placed inside a magnetic field of an inductor. Various forces acting on the magnet cause its misalignment. We will look into these forces and use them for the description of the observed phenomena.

The main purpose of the experiment was in switching on and off the current into the inductor. Using correct intervals of switching we find resonant frequency of the pendulum. This will lead to maximizing of the misalignment of the pendulum and the participant of the experiment will be able to “catch” a magnetic object situated near the location of the maximum alignment with the magnet.

Figure 2: Electromagnetic pendulum (dismantled)

Figure 1: Electromagnetic pendulum

Goal of the experiment

Due to the interactive nature of the experiment it has substantial potential to captivate students. It might lead to easier strengthening of the knowledge that electricity and magnetism are interconnected phenomena which is also the main message of this experiment. In higher grades it is possible to connect electromagnetic with mechanical phenomena such as resonant frequency of pendulum, effects of forces and the moment of forces, decomposition of forces etc.

Components and their cost

	Components	Cost [Czech crown]
1	AA battery (3 pcs)	30
2	Holder for batteries (1 pc)	15
3	Doorbell switch (1 pc)	80
4	Isolated wire (1 m)	5
5	Lacquered wire (0.7 mm, 5 m)	50
6	Neodymium magnet (cylinder, 2 pcs)	20
Total		**200**

Instructions for fabrication and description of the construction

First step in construction of this pendulum is fabrication of the coil with the diameter of ca. 7 cm. On a bottle, e.g. of wine, we wind ca. 30 coils with lacquered wire. We take the coil and carefully take it off the bottle and tape it together e.g. with tape or glue gun so that it doesn’t unwind. Next we glue the coil in two opposite points of the circumference to two wooden laths. We drill a hole in each of them at the same distance from one end and we insert a small metal shaft in one of them. We firmly attach it in the opening (e.g. with glue gun or glue). The second end of the shaft will be lose which will make it easier to slide it into the second lath. We firmly attach the bottoms of the laths to the underlay. Next we make an opening in the wooden stick (instead of the wooden stick a straw can be used, which is more accessible and adjustable) so that it will have slightly bigger diameter than the diameter of the shaft. Great deal of friction will be generated while the stick turns and also the stick won’t be wobbling in the bigger opening. It is important that the opening is at such distance from the bottom of the lath so that the end of the stick after inserting the shaft into the opening is c.a. 5 mm above the centre of the coil. We insert the stick onto the shaft and over its end we attach two small neodymium magnets (in our case cylinders), thus creating one magnet. It is important that the south-north axis of the magnet is horizontal and also perpendicular to the rotation axis. We connect the ends of the coil using the doorbell switch to the three batteries of type AA in series.

As an appealing element we can additionally install for example a metal pendant which students can try to “catch”.

Technical notes

We can see from the picture that in our construction yellow plastic pieces were used under the ends of the wooden sticks. These are four parts of a Lego kit where one “cube” is always firmly placed to the underlay and the second one is firmly attached to the wooden stick. According to this setup sticks can be taken off and laid down which is a huge advantage during moving the experiment on longer distances.

It’s worth mentioning the diameter of the lacquered wire. The question is if the wire could be thinner – two batteries cannot possibly provide a big enough current that the wire would heat up to critical temperature. The reason why the wire with such diameter (0.7 mm) was used is purely practical. After unpinning the sticks the coil from the thinner wire could be easily deformed during transportation of the equipment while the wire with the mentioned diameter makes the coil more robust and resistant.

Physics of the experiment and its process

Our main goal is to describe why, how and with what force the pendulum with magnets as a weight will displace itself from its resting position inside the magnetic field of the coil. Physics of the experiment is not trivial therefore we will analyse it gradually and in detail by means of the following aspects. First we will focus on basic effects which could happen in placing a magnet into a magnetic field. Next we will focus on an easier version of the experiment and then we will apply the results on our experiment.

Description of behaviour of the cylindrical magnet in magnetic field

If we place a free, axially magnetised (standard direction of the magnetisation) magnet into a homogenous magnetic field it will turn in the direction of the magnetic induction lines.

If we place the magnet into an inhomogeneous field (e.g. field of another magnet) except from turning of the magnet there will be another effect – pulling of the magnet into a spot with higher density of the lines of induction (into stronger field) or (with opposite orientation of one of the fields) expelling of the magnet from the spot with higher density of lines of induction. These phenomena are the reason why two magnets either attract or repel each other.

Now we will move on to our experiment. As a source of an inhomogeneous magnetic field we will have a coil with current. Magnet is magnetized axially but in contrast to the previous idea it is firmly fixed to the end of the wooden stick which is loosely hanged on its upper end which does not allow the magnet to orientate itself in to the direction of the lines of induction or be pulled into the coil. We will try to find out why the magnet with the stick swings. In other words we are asking what forces act on the pendulum and what is the resultant force. Rather than on quantitative conclusion we will focus more on the qualitative conclusions such as in what direction and why the pendulum swings.

Analysis of the forces and moments having an effect on the magnet in magnetic field (Figure 3)

We consider the pendulum – horizontally orientated magnet (cylinder) length d – which is fixed in half of its length to the fixed hinge (e.g. to wooden stick) with length l. We will place a barrier to the left side of the weight so that the pendulum won't be able to displace itself to that side. In that case the pendulum won't be moving and during analysis of the situation we will take advantage of the fact that the final force of the forces must be zero. We assume that the magnet is to be found in a *homogenous* magnetic field oriented (without loss of generality) vertically from bottom up.

As we said magnet *is trying* to orientate itself in the direction of the field – for simplicity let's imagine that the force acts downwards on one pole and upwards on the other. Now we consider only the force acting down. Instead of the magnet and homogenous field we will also imagine cylindrical weight and force $\overrightarrow{F_{1_A}}$ with the point of action of the force in point A (outer point of the weight on its axis) pointing vertically down.

We will gradually analyse forces which act on different parts of the pendulum. The gravitational force $\vec{F_g}$ acts on the pendulum which is compensated by the force of the hinge $\vec{F_z}$. We don't need to consider these forces further. At point A we act vertically down with the force $\overrightarrow{F_{1_A}}$ which is compensated by force $\overrightarrow{F_{1_D}}$. Weight on entanglement acts with a force $\overrightarrow{F_{2_C}}$ but thanks to the Newton's third law of motion (Action and reaction) entanglement also acts on the weight with equal force $\overrightarrow{F_{3_C}}$ but in opposite direction. Besides these forces acting on the pendulum there is also the force of the point of attachment $\overrightarrow{F_{5_D}}$ and on the point (again by the 3. Newton's law) acts opposite force of the stick $\overrightarrow{F_{4_D}}$. Therefore the weight is not moving – the forces are balanced so that the resultant force is zero.

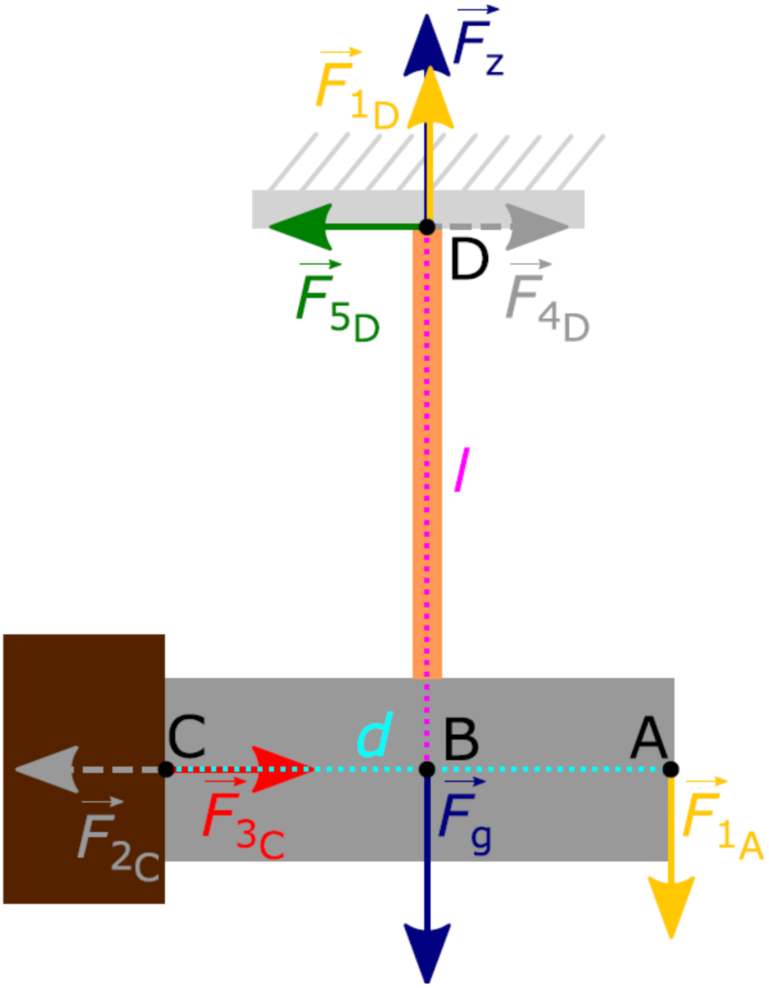

Figure 3: Forces acting on the pendulum

Now we will analyse the moments which act on the pendulum. Moment $\vec{M}$ can be determined generally from this formula:

$$\vec{M} = \vec{r} \times \vec{F},$$

Where $\vec{r}$ is an arm of force $\vec{F}$.

We can use formula $M = rF\sin\alpha$ to get the absolute value of moment $\vec{M}$ where α is an angel between the arm of force $\vec{r}$ and force $\vec{F}$.

According to point B (centre of the weight) the value of force $\overrightarrow{M_1}$ is:

$$M_1 = \frac{d}{2} F_{1_A} \sin\alpha.$$

Angle α is 90° because the force $\overrightarrow{F_{1_A}}$ and its arm of the force with length $\frac{d}{2}$ are perpendicular therefore $\sin\alpha = 1$ and $M_1 = \frac{d}{2} F_{1_A}$.

The value of moment $\overrightarrow{M_2}$ (which acts in the opposite direction than $\overrightarrow{M_1}$) can be calculated from

$$M_2 = lF_{3_C} \sin\beta.$$

Angle β is 90° because force $\overrightarrow{F_{3_C}}$ and its arm of the force with length l are perpendicular therefore $\sin\beta = 1$ and $M_2 = lF_{3_C}$. The absolute values of the moments must be equal because the pendulum is not moving. Therefore

$$\frac{d}{2} F_{1_A} = lF_{3_C}.$$

From this equation we can express the absolute value of force F_{3_C} as

$$F_{3_C} = F_{1_A} \frac{d}{2l}.$$

If we remove the barrier the pendulum will swing thanks to force $\overrightarrow{F_{3_C}}$. Now let us get back to the situation where the weight is a magnet in a homogenous magnetic field. Because of that force $\overrightarrow{F_{1_C}}$ also acts on the weight – magnet – vertically down at point C. Its absolute value will be equal to force $\overrightarrow{F_{1_A}}$ but it will have the opposite direction. After all we come to the conclusion that its effect on the entanglement has the same direction and the absolute value as the effect of force $\overrightarrow{F_{1_A}}$. The final absolute value of force $\overrightarrow{F'_{3_C}}$ acting on the pendulum will be:

$$F'_{3_C} = F_{1_A} \frac{d}{l}.$$

Now we will do an experiment with the coil through which a current is flowing and with a little magnet on the stick. We will be moving the pendulum from the height *h* above the centre of the coil vertically down to the height -*h*. We observe that the force acts on the magnet in direction to the one side of the coil in the half-plane above the coil. In the half-plane under the coil the force acts on the other side. We cannot explain this phenomena from the previous considerations. The reason for that is the inhomogeneous field of the coil that we have not considered. We will enlist the help of the theory of "magnetic quantity".

Description with "magnetic quantity" [1] (Figure 4)

The main idea of the theory "magnetic quantity" [L. Dvořák, 2016] lies in imagining the ends of a magnetic pole as areas with magnetic quantities Q_{m_1} and Q_{m_2} which characterize "how strong the magnetic poles are". For calculation of the magnetic force F_m between the poles of the long stick magnets we can use the following formula (analogous to the Coulomb's law):

$$F = \frac{1}{4\pi\mu_0}\frac{Q_{m_1}Q_{m_2}}{r^2}.$$

In electrostatics we calculate the force acting on a charge in an electric field as $F = QE$. In magnetic field we count the "force on the magnetic quantity" as $F = Q_m H$, where H is the magnetic field intensity. Combining these two formulas we get the magnetic field in around the pole of the long stick magnet

$$H = \frac{1}{4\pi\mu_0}\frac{Q_m}{r^2},$$

where Q_m is the magnetic quantity of the particular pole. Magnetic field intensity and magnetic induction are related as $B = \mu_0 H$.

Force F acting on the pole of the stick magnet is therefore

$$F = \frac{BQ_m}{\mu_0}.$$

The magnet in our experiment does not have the shape of a stick therefore this formula does not work on it. We can make the magnet longer and then we get a reasonable approximation.

In the magnetic field of the coil the forces $\overrightarrow{F_N}$ and $\overrightarrow{F_S}$ act on the magnetic quantity at the ends of the magnet. Their direction is tangential to the induction lines of the magnetic field of the coil. The most important forces for us as are $\overrightarrow{F_{N_X}}$ and $\overrightarrow{F_{S_X}}$ which are the projection of the forces $\overrightarrow{F_N}$ and $\overrightarrow{F_S}$ to the x-axis. Forces $\overrightarrow{F_{N_X}}$ and $\overrightarrow{F_{S_X}}$ point in the same direction so it is clear that the final force acting on the magnet will go left.

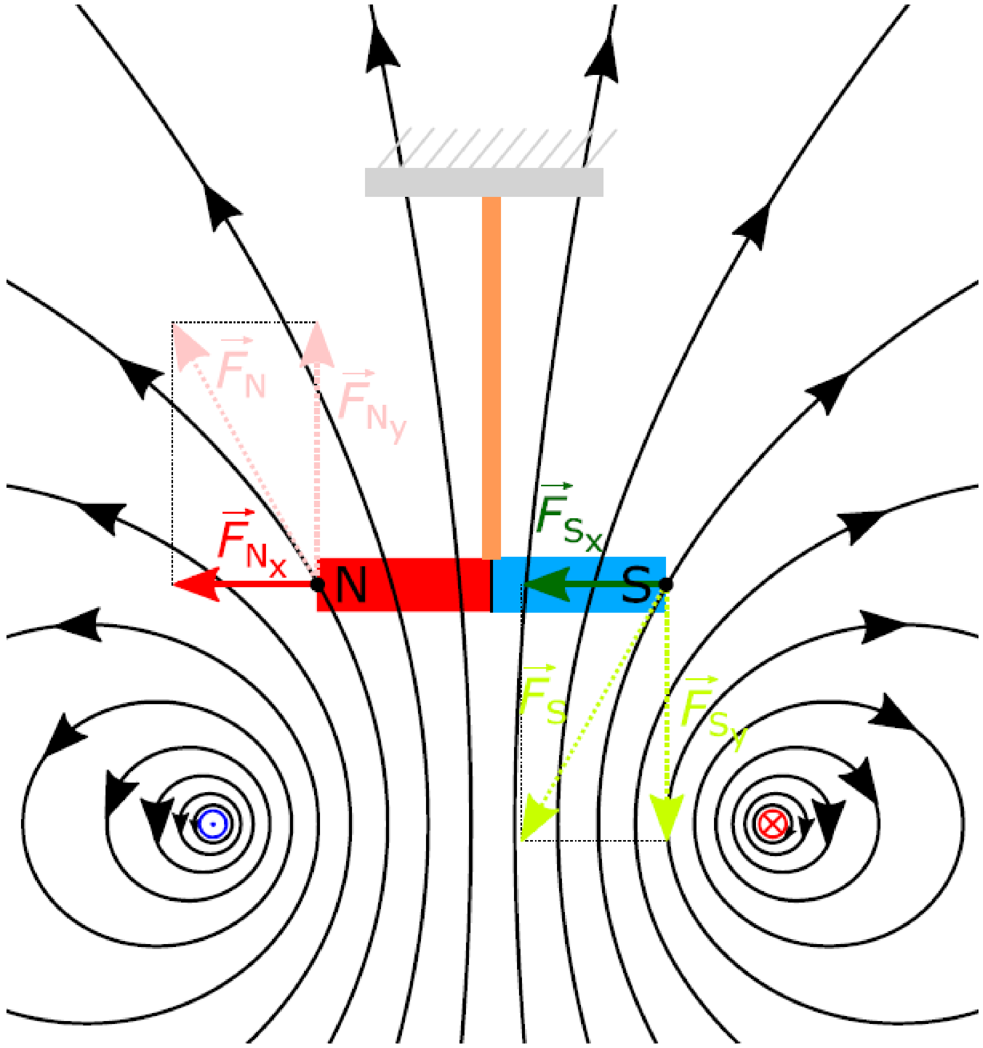

Figure 4: Forces acting on the magnetic quantity

Conclusions and the integration into teaching

With the help of previous analysis we qualitatively described why the pendulum is displaced. Let's remember that we used various approximations and that we omitted some effects. For example when the magnet leaves its rest position the situation becomes greatly asymmetrical and new forces appear to be acting on the pendulum that were not applied in the rest position. For example the magnet will be pulled into the spot with higher density of induction lines as we commented previously.

It is a matter for consideration which description of this phenomena to use while teaching it to the students. Analysis of the forces and the moments acting on the magnet in the magnetic field is rather hard but familiar to students from their mechanics classes. Description with magnetic quantities can be unusual for students but for the basic understanding it is probably easier. Be careful of students gaining impression that there are some "magnetic charges" located at the ends of the magnet. Integration of this experiment into teaching can be interesting for many students because they can try catching the pendant which is a very popular activity. In the case where students will be playing with the pendulum we must take into consideration the time consumption as students usually do not catch the pendant even after couple of minutes. The interesting fact is that the students who usually catch the pendant are the ones who play some musical instrument, dance or some other activity which requires the sense of rhythm. One who lacks this ability struggles a lot while trying to catch the pendant. The reason is that the switch must be pressed in right intervals and that is the privilege of people able to feel the rhythm.

Literature

[1] DVOŘÁK, Leoš, 2016. *O magnetu, magnetických tělesech a velikém magnetu Zemi* [online]. [cit. 1. 10. 2018]. Available online: http://kdf.mff.cuni.cz/vyuka/Fyzika2elmag/OmagnetuMagnetickychTelesech_DilnyHeureky2016.pdf