Unconventional experiments with coupled oscillators

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Introduction

The understanding of the dynamics within coupled oscillators is crucial for the explanation of the transition from the behaviour of an isolated oscillator to that of a series of oscillators interconnected by a coupling mechanism, through which waves propagate. Of particular importance is the observation of energy transfer between oscillators. Conventional analysis is typically restricted to the examination of coupled pendulum oscillations. However, less emphasis is placed on the examination of energy transfer between electromagnetic oscillators.

Experiments with coupled electromagnetic oscillators in the past

One of the authors of this contribution studied the vibrations of inductively coupled oscillators using the instruments available at the time in the 1960s [1]. The resonant curve was measured point by point using a tone generator and a cathode-ray voltmeter (Fig. 1a). The oscillations were excited at intervals corresponding to the frequency of the time base of the oscilloscope by a simple relaxation generator of sawtooth waves and the oscillograms of vibrations were obtained by photographing the oscilloscope screen (Fig. 1b).

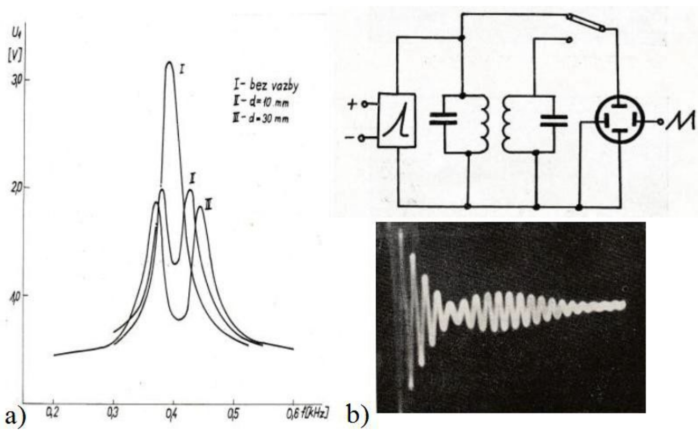


Fig. 1

The two maxima of the resonant curve correspond to the fact that the events in coupled oscillators can be interpreted as a superposition of symmetric vibrations (both the oscillator and the resonator oscillate with the same phase) and antisymmetric vibrations (the oscillators oscillate with opposite phase), which leads to the creation of characteristic beats. Their period depends on whether the coupling between the oscillators is tight or loose. In the explanation, we can apply analogies of mechanical and electromagnetic vibrations, described by second-order differential equations.

Coupled mechanical oscillators	LC circuits with inductive coupling
$m\frac{\mathrm{d}^2y_1}{\mathrm{d}t^2}+b\frac{\mathrm{d}y_1}{\mathrm{d}t}+ky_1+c(y_1-y_2)=0$ $m\frac{\mathrm{d}^2y_2}{\mathrm{d}t^2}+b\frac{\mathrm{d}y_2}{\mathrm{d}t}+ky_2-c(y_1-y_2)=0$	$L\frac{\mathrm{d}^2q_1}{\mathrm{d}t^2}+R\frac{\mathrm{d}q_1}{\mathrm{d}t}+\frac{q_1}{C}+M\frac{\mathrm{d}^2q_2}{\mathrm{d}t^2}=0$ $L\frac{\mathrm{d}^2q_2}{\mathrm{d}t^2}+R\frac{\mathrm{d}q_2}{\mathrm{d}t}+\frac{q_2}{C}+M\frac{\mathrm{d}^2q_1}{\mathrm{d}t^2}=0$

The last term of the equations expresses the force or electrical voltage of the mutual interaction of the oscillators. In mechanical oscillators, it depends on the coupling coefficient *c* and the difference of the instantaneous amplitudes (coupling by amplitude). In electromagnetic oscillators, it is the induced voltage, which depends on the coefficient of inductive coupling *M* between the coils and the second derivative of the capacitor charge. However, the above differential equations are beyond the scope of direct use in secondary education. They are the basis of dynamic models, which are used to clearly interpret the relatively complex vibrations of coupled oscillators.

Exploring novel methods for demonstrating the behaviour of coupled oscillators

Contemporary IT capabilities provide new means for studying events in coupled oscillators. We will present several examples that can be used as a topic for independent student work

- **Video-analysis of oscillations of coupled pendulums**

There are various programs available for the video analysis of mechanical movements, as described for example in [2]. Currently, the program Tracker [I] is considered to be the most appropriate. The recording of vibrations of coupled pendulums by PHYWE was analysed. An image recording lasting 1 minute was taken with a digital camera for this purpose. With a scanning frequency of 25 frames/s, this results in 1,500 frames with a time step of 0.04 s. The outcome of the video analysis is evident from Fig. 2.

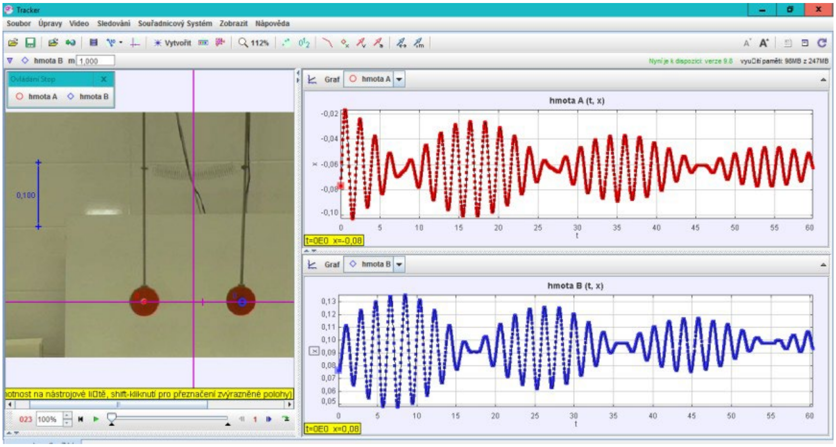


Fig. 2

- **Recording oscillations of coupled pendulums with a Vernier sonar**

A similar time-diagram of the oscillations of coupled pendulums was obtained using two position and motion sensors, Motion Detector 2. To prevent interference of the simultaneous recording of the oscillator and the resonator, a partition was placed between the pendulums (Fig. 3). The obtained data was analyzed using the program Logger Pro [II] (Fig. 4).

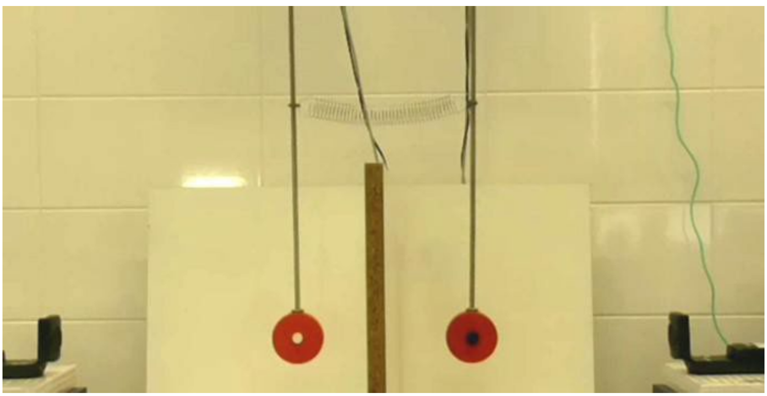


Fig. 3

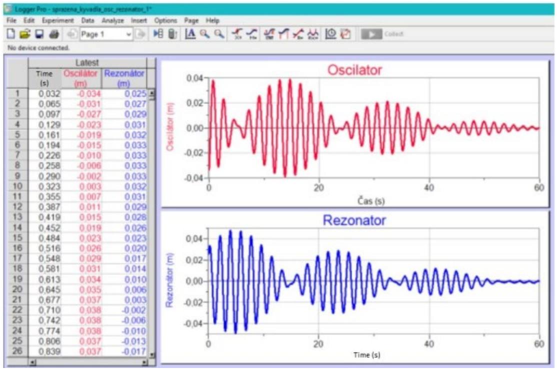


Fig. 4

- **Recording oscillations of magnetically coupled elastic bands**

In the curriculum on oscillations, the spring oscillator is given preference over the pendulum. Demonstrations of coupled spring oscillators are difficult, and in teaching we limit ourselves to computer simulations. However, we can easily demonstrate elastic oscillations of a system of two steel belts, approximately 30 cm in length, placed at a mutual distance of about 3 cm. The coupling is provided by two ferrite magnets attached to the ends of the belts so that they repel each other (in more detail see [3]). The mass of the magnets also affects the frequency of the belt's oscillations. Near the magnets, there are coils from a step-down transformer (600 turns, I-shaped core). The voltage induced in the coils allows, with the help of connected voltage sensors (DVP-BTA), to obtain data for processing with the program Logger Pro (Fig. 5).

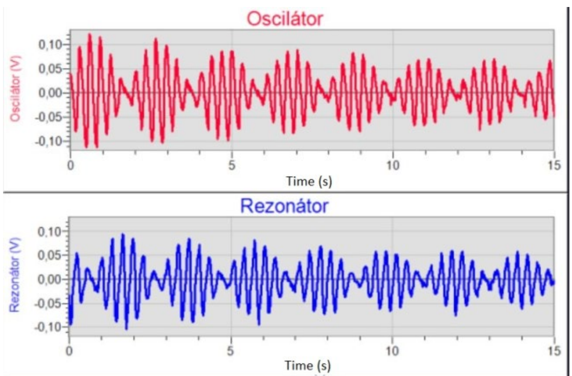
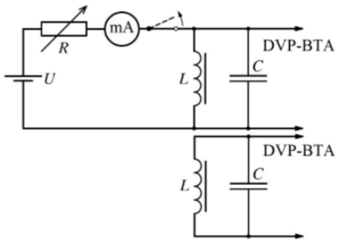


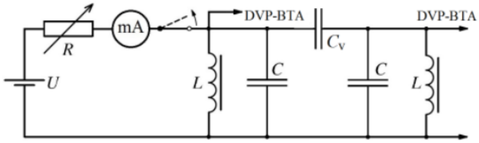
Fig. 5

- **Oscillations of coupled electromagnetic oscillators**

The experiment used two LC oscillatory circuits consisting of 600-wind coils with a I-shaped core and 1 μF capacitors. The coils are located on a common axis such that the air gap between the core of the coils is adjustable with a width of approximately 4 cm. The method of excitation of oscillations is shown in Fig. 6 (further details can be found in [4]). The obtained data was again processed by the Logger Pro program (Fig. 7). The experiments were performed both with inductively and capacitively coupled circuits (the coupling capacitor C_v also had a capacity of 1 μF).



Inductive coupling



Capacitive coupling

Fig. 6

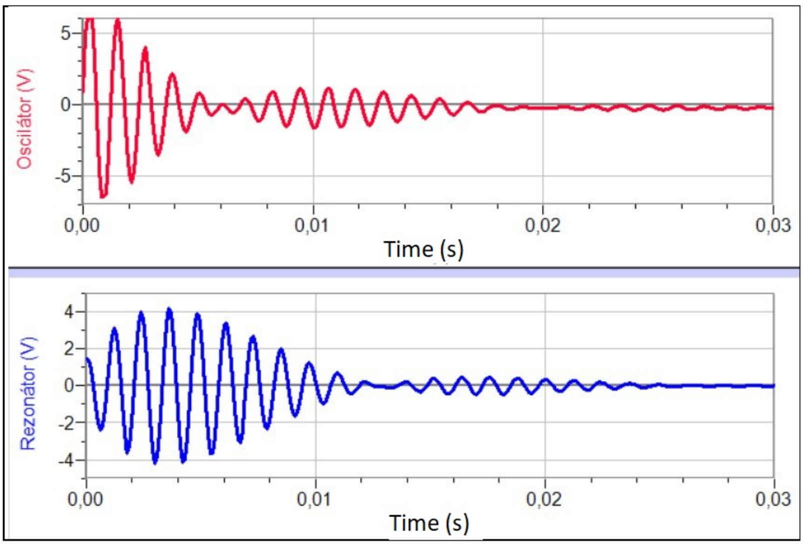


Fig. 7

- **Models of oscillations in coupled oscillators**

Models of both mechanical and electromagnetic bound oscillators were created using the program Modellus 4.01 [III] (Fig. 8). The models are described in [3].

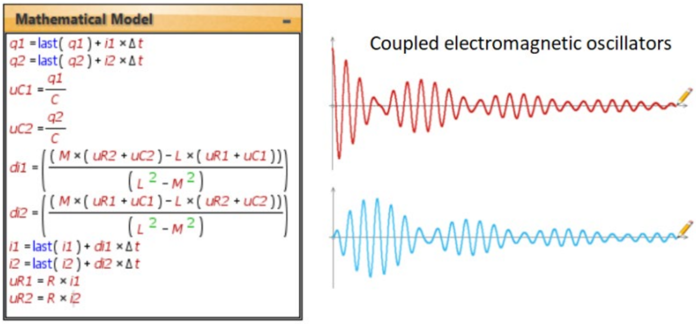


Fig. 8

More complex models of coupled oscillators can be created using the simulation program NL5 Circuit Simulator [IV]. This program allows you to easily create a circuit diagram by drawing with the mouse on the monitor screen and immediately display the time-domain and frequency-domain characteristics of the oscillations in the circuit. This program was used to simulate all the configurations of coupled electromagnetic oscillators (see for example Fig. 9). A series of capacitively coupled oscillators was also created, whose frequency characteristics showed a corresponding increase in resonance maxima (Fig. 10). This allows for extrapolation to a long chain of coupled oscillators, i.e. a two-conductor transmission line with a progressive electromagnetic wave.

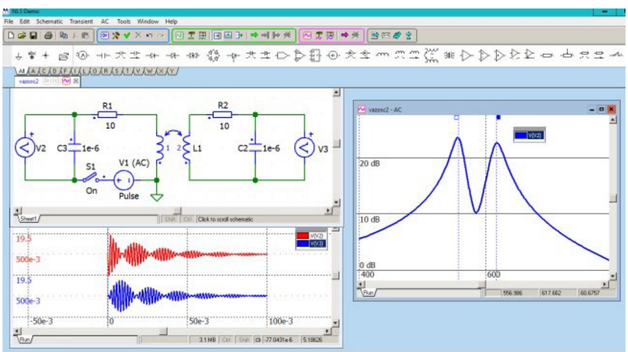


Fig. 9

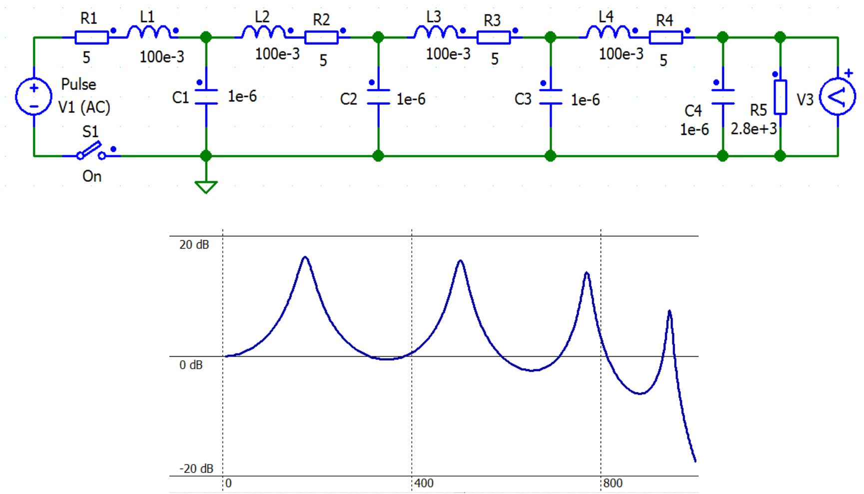


Fig. 10

Literature

Lepil, O.: *Příspěvek k metodice výkladu rezonančních jevů ve vázaných oscilátorech*. In: ACTA UP, Fac. Rer. Nat. Tom 15, 1964. Dostupné na: http://dml.cz/dmlcz/119814

Lepil, O.: *Videoanalýza fyzikálních dějů*. In: Veletrh nápadů učitelů fyziky IX, sborník z konference, PdF MU Brno 2004, ISBN 80-7315-084-0. Dostupné na: http://vnuf.cz/sbornik_old/Veletrh_09/09_07_Lepil.html

Lepil, O.: *Demonstrujeme kmity netradičně*. Prometheus, Praha 1996.

Lepil, O., Richterek, L.: *Dynamické modelování*, Repronis, Ostrava 2007. Dostupné na: http://ufm.sgo.cz/ke_stazeni.php

Lepil, O., Látal, F.: *Experiment v učivu o kmitání elektromagnetického oscilátoru.* MFI roč. 22 (2013), č. 5, s. 344. Dostupné na: http://mfi.upol.cz/files/2205/mfi_2205_344_354.pdf

Software

Tracker 4.97: <http://www.cabrillo.edu/~dbrown/tracker/>

Logger Pro 3.8.2: <https://www.vernier.com/>

Modellus 4.01: <http://modellus.fct.unl.pt/>

NL 5 Circuit Simulator: <http://nl5.sidelinesoft.com/>