More ideas from Malá Hraštice, this time with water

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Abstract

The article describes several experiments with water: 1) the effect of water's diamagnetism on the water level above magnets, 2) the apparent depth of objects submerged in water from different observing angles, 3) easy method for measuring the viscosity of water, and 4) demonstrating projectile motion of water using a plastic syringe and their connection to the continuity equation.

Introduction

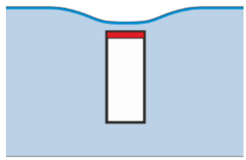
Over the time there have been many articles presenting ideas that stemmed from spring camps for future physics teachers organized by the Faculty of Mathematics and Physics of the Charles University. These camps have been taking place for 20 years now, more information can be found on the camps website [1]. This year, water (i.e. water and air) was the theme of the spring camp's mini-projects, so there was an opportunity to try out experiments that I had only seen or had tried qualitatively in the past.

You can consider the experiments described below as a suggestion for extending your teaching (or perhaps more suitably in elective seminars or class projects) or simply just as experiments that can broaden our own understanding of relevant physical phenomena. At least I personally have the impression that I have learned a lot during the realization and interpretation of these experiments. So if you become interested or get the feeling that you as physics teacher have learned, remembered, or thought of something new, this article will have achieved its intended purpose.

Article [2] that I wrote after my return from Malá Hraštice gives a testimony how I battled the development and realization of the experiments. This text is briefer, though it describes some experiments a tad bit more thoroughly.

Water and magnets (water's diamagnetism)

I have seen the following experiment with water and a magnet done by prof. Planinšič at the University of Ljubljana; it's thoroughly described in the article [3]. However, it is rather simple: put a strong neodymium magnet into the water in such a way that its one pole will be very slightly situated below the water level (less than a millimetre). Because water is diamagnetic, the water level above the magnet's pole will lower a little bit, there will be a little "dent".



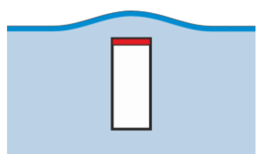
The fact that the water level above the magnet decreases is at first glance surprising for many people. Shouldn't it, on the contrary, increase thanks to the repelling from the magnet? We might understand the situation better if we think about the energy: a droplet of water has higher energy near the pole than when it's further from it (because it's being repelled, we must push towards the magnet). The energy of the droplet is equal to the sum of its potential energy in Earth's gravitational field and its energy in the magnetic field. The energy on the water surface is everywhere the same because otherwise, the water would go to a place with lower energy. Thus, when the energy of the droplet is higher near the pole of the magnet, its potential energy in the gravitational field must be lower therefore the water level must be lower.

Photography shows that it really is the case. However, the change is barely visible. Theoretical derivation (shown in [2]) gives this formula for the change in height

$$\Delta h = \frac{B^2|\chi|}{2\mu_0 \rho g}$$

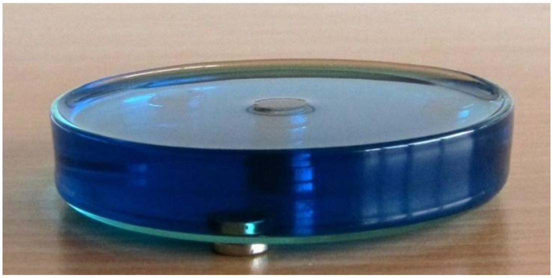
Where B is magnetic induction near the pole of the magnet and χ is the susceptibility of water (approx. $-9 \cdot 10^{-6}$); symbols for the permeability of a vacuum, density of water and gravitational acceleration are obvious. Magnetic induction near the pole of the magnet is equal approximately to 0.5 T (verified by measurement). The change in height is then only about 0.1 mm. In article [3] its authors measure the change of height in a somewhat complicated way by using the reflection of a laser beam on the edge of the "dent", where the water level is sloped; then the slope must be integrated in order to get the total Δh. In our case I tried to measure the change by observing the reflection of a thin stick right above the water level, the results are however rather inaccurate and the method must be improved in the future. So let us consider the above-mentioned experiment only as a qualitative demonstration for now.

But what if we use a liquid that is paramagnetic? It is attracted to the magnet, so we should observe a little bump. The paramagnetic liquid is for example copper sulfate, so why not try the experiment with its solution?



Rough theoretical estimations for this saturated solution give an increase in height of about 0.2 mm.

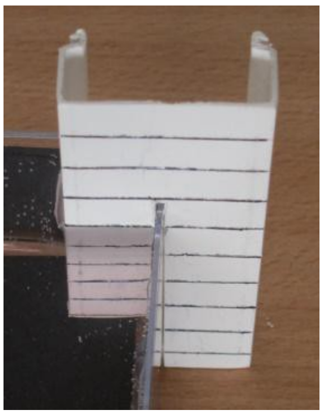
The experiment shows that the water level above the magnet actually increases, although only by a small portion.



A look through the water (apparent depth)

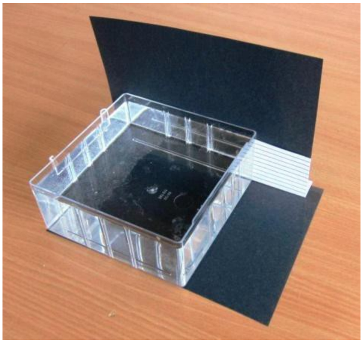
When we look through the water at objects their depth as we perceive it looks smaller than it really is. It is commonly said that the apparent depth is reduced at 1/n, where n is the refractive index of water. That's true however only when we look perpendicularly or only a little bit off at the water level.

The apparent reduction of depth can be demonstrated for example with a little tablet that we cut in such a way that it can be slid onto the perpendicular side of a glass of water. We draw parallel lines on the tablet that are 1 cm away from each other. On the side that's under water lines seem to be closer together than on the opposite side that is in the air. We can also quite accurately measure the apparent reduction of depth.



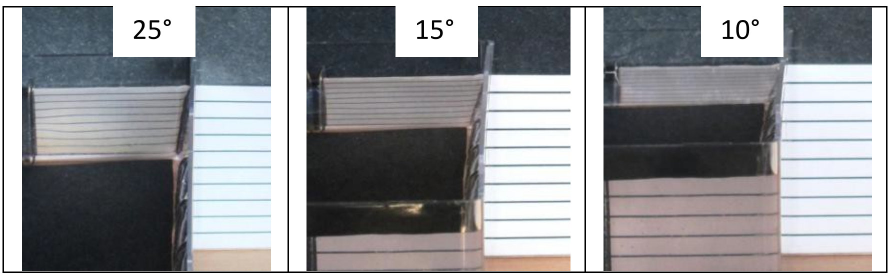
Photography on the right shows a look that differs from the vertical direction at about 30°. One of the lines blends with the water level. When we compare the widths of stripes on the left and the right we see that the apparent depth is reduced in the 2/3 ratio, which is a little bit more than the ratio 1/n would suggest (1/1.33 = ¾). If we observe our "line ladder" from angles close to the perpendicular one (90°) the apparent reduction truly approaches the 1/n ratio, but when we choose the observation angles to be more and more skewed, the apparent reduction of depth will be more and more distinct. For that matter, we all probably know this effect from swimming pools, if we put our head close to the water and look in an almost horizontal direction.

We can examine this effect also quantitatively and only by using very simple tools. We pour water into a cuboid-shaped container, let's call it a box. In our case we used a homemade plexiglass box, of course, we could also use an aquarium.

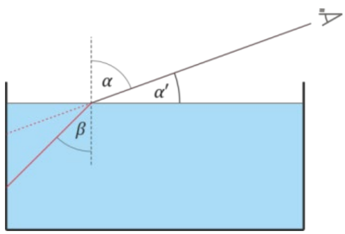


We put paper to the backside of the box with parallel lines (in our experiment they were 0.5 cm apart). It is recommended to also cover the back and bottom side of the box with dark paper so that light and its reflections don't interfere with our observations. The experiment itself is designed in such a way that we or our students realize what all possible factors can influence and possibly harm our observations or even our attempts at photographic documentation.

The following pictures show how observations at angles formed by horizontal direction and the camera.



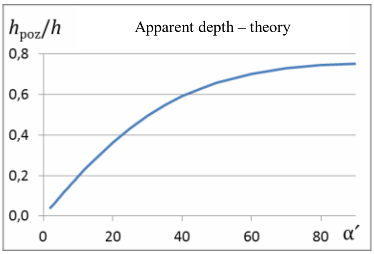
Let's just remark that in the left bottom corners of the middle and the right picture the visible lines are observed through the front side of the box – if students were confused by this, cover also the front side of the box with dark paper.

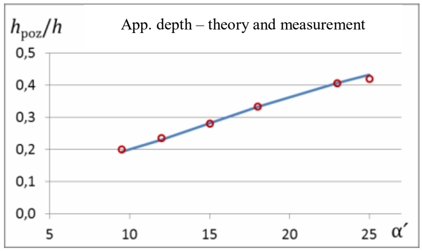


For apparent (observed) reduction of depth we can quite straightforwardly derive theoretical formula

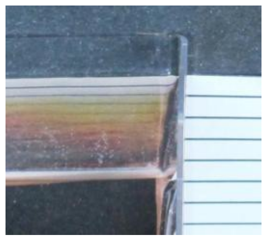
$$\frac{h_{\mathrm{poz}}}{h} = \frac{\sin\alpha'}{\sqrt{n^2 - 1 + \sin^2\alpha'}}$$

The following graphs show the ratio of apparent and true depths. The left one shows the ratio as a function of the observation angle (refractive index of water n = 1.33) and in the right graph, we see quite good correspondence between measurement and theory.



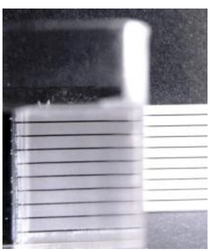


We could calculate the refractive index based on our measurements. However, this method surely is not the one that's the most suitable. During the experiment, we had a great problem with reaching the sufficient precision of measurement of angle α'. Soon after it turned out that it's best to measure the angle between the axis of our camera and the horizontal direction, i. e. the angle between the camera's display and the vertical direction; all that utilizing simple tools using protractor and a lead bob (e. g. Small nut on a thread).



We can't measure bigger angles than approximately 28° with our equipment described above. This is due to the total internal reflection (TIR) occurring on the inner side of the box. The right picture shows that the angle at which point the total reflection is observed slightly differs for various colours of light: we see rainbow coloration on the back side of our box.

If there is black paper on the bottom of the box, the backside will appear dark during total reflection. If there is none, the backside will appear shiny.



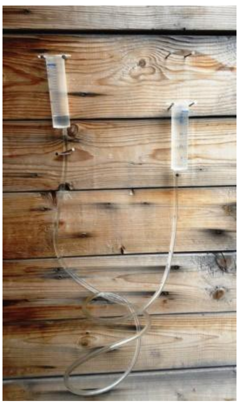
We can see similar effects on a larger scale when, for instance, observing aquaria. Let us notice one more phenomenon though, that can be observed and measured even with our water box experiment: look horizontally at the lined card through the front side of the box (where water is) and simultaneously watch the card through the air and focus on the distance between the lines. The stripes observed through water are further apart than the ones seen through the air. A simple explanation can be likewise based on the apparent depth, or rather the apparent length between the front and back sides of the box. In water, this length is shortened in ratio 1/n which means approximately 3/4 of the true length. Therefore (if our eye is close to the front side) the stripes observed through water are about 1.3 times wider than the stripes observed through the air.

Water flow in a hose (determining the viscosity of water)

When water flows through a pipe or a hose the hose exerts some resistance against the flow which is proportional to its viscosity η. If the flow is laminar then the "pressure losses" (the difference Δp of pressures on both ends of the hose) is determined by Poiseuille's law (e. g. [4]):

$$\Delta p = \frac{8\eta l}{\pi R^4}\frac{\Delta V}{\Delta t},$$

where l is the length of the hose, R is its radius and ΔV is the volume of water that flows through the hose in time Δt. Poiseuille's law certainly does not belong to secondary curricula however, it is most definitely interesting to at least qualitatively entertain the idea that the hose indeed resists the water flow and ponder the extent to which this resistance is dependent on the radius of the hose. (For example, if your water pipe shrinks due to the poor maintenance by some further unspecified algae to one half of the original, then you will experience that under the same pressure the flow will decrease sixteenfold!)

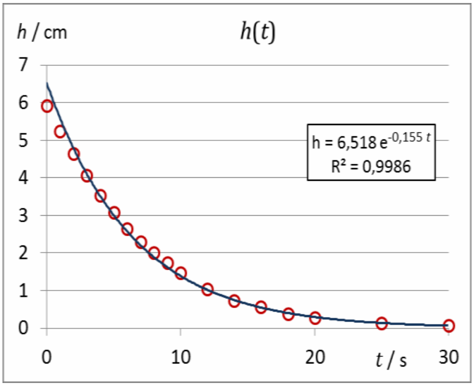


We can also use the aforementioned formula while measuring water viscosity. This rather simple experiment utilizes two plastic syringes (more precisely their external parts). We can tightly attach little plastic hose with inner diameter of 4 mm to their "nozzles". If we fill the two interconnected syringes with water and lift one of them, water starts to flow from the higher to the lower one. Water flow is according to the Poisseulle's law proportional to the pressure differential (and as such also the height difference between two water levels). The flow is also proportional to the speed of change of the height of the slope of water h in the syringe. Because $\Delta V = S\Delta h,$ so then

$$\frac{\Delta V}{\Delta t} = S \cdot \frac{\Delta h}{\Delta t}.$$

We could derive that the gauge height decreases exponentially with time, although this is more suitable for advanced classes for interested students in secondary school. In the introductory university level or for those equipped for the task we could even derive the relevant differential equation for an infinitesimal change in height:

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{\rho g R^4}{4\eta l R_{\text{syringe}}^2} h$$



We plotted the following graph with values acquired from a video record. It shows that the decrease in height is quite similar to an exponential. When we compare the solution to the equation

$$h = h_0 \cdot \exp\left(-\frac{\rho g R^4}{4\eta l R_{\text{syringe}}^4} t\right)$$

and the measured values then we can compute η.

The measured value is approximately $\eta = 1.2 \cdot 10^{-3}\ \mathrm{Pa \cdot s}$, so it's roughly 30 % more than the common table value $0.9 \cdot 10^{-3}\ \mathrm{Pa \cdot s}$ (for temperature ±25 °C, more in [5]). It is actually quite interesting to investigate the cause of such a big deviation. Of course, there could be resistance inside the nozzles of the syringes at play or the fact that the flow isn't perfectly laminar.

Let us further notice that viscosity can be measured even without the knowledge of the differential equation, simply by the average difference of heights and average speed of flow. The additional deviation caused by this simplification is of the 10 % magnitude.

How far can a syringe shoot (vertical and projectile motion of water and the continuity equation)

The following experiment was already briefly mentioned in [6]. The question, in this case, is how high we can shoot water with a simple plastic syringe (in our case the volume was 20 ml) and how this height depends on the speed of the piston of our syringe.

As the photo show (and how the participants of this demonstration could see live) the water reaches the height of approximately 5 meters. From this and a commonly known secondary school formula $v = \sqrt{2gh}$ the corresponding speed equals plus or minus 10 m/s. We get practically the same speed if we shoot water horizontally and use the according formula for horizontal motion.

Does this correspond with the speed of the piston of our syringe?

The diameter of the nozzle is 2 mm, the inner diameter of the syringe is 2 cm, which is 10 times bigger. The ratio of the cross-sections is 100:1. The continuity equation tells us that the speed of the piston is 0.1 m/s, which is 10 cm per second. And really, we empty the full syringe (water column is approx. 6 cm) in about a second.

We could increase the precision of the experiment. Measure the speed of the piston with a camera, there's always room for improvement. Another option is to change the diameter of the nozzle. The simplest way would be to attach a little part of a small plastic hose to the nozzle. Twice bigger diameter means four times bigger cross-section, that is four times slower water which makes 16 times lower height of the peak, all that under the assumption that we move the piston at the same speed. We could achieve the same piston speed by engineering some precise machine to move it... As I said, there is an immense number of ways how to improve, upgrade and tinker with all the aforementioned experiments either in form of group projects, practical assignments, etc. Hopefully, we will reach such ideas of improvement and why not in the next Physics Teachers' Inventions Fair.

Acknowledgements

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