Physics and Sport

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1. Introduction

I will start with some arguments why topics of sports could/should be included in physics teaching at several school levels up to university.

The first one is **interdisciplinarity**. I think there does not exist any curriculum all over the world, in which it is not stated that one should teach and work interdisciplinary. That one should try to combine at least two subjects on a certain topic. Physics and sports could be a good example. Also because a collaboration is sometimes obligatory, for instance when a physics lesson takes place in a swimming pool, since a physics teacher is not allowed to supervise such sports events.

Motivation: If students are asked which school-subject they like most or not so much, sports is probably on one end, physics at the other. So why not take over some of the motivation for sports in the physics class.

Activity: One of the most attractive features of sports is the activity. Young persons want to be active, to move, to run. The occasion to be active in a physics class is quite rare. The most activity is probably in connection with experiments. One can take a sports activity as an experiment with the own body performing measurements on it.

Modelling: Modeling is important for all natural sciences. We always work with models. Students do not see this so clearly. Sports actions, in particular when the human body is involved, are very complex. In order to describe and explain them, one has to work with simplifications, with more or less sophisticated models. Working on such examples, students understand the necessity and the benefit of models.

Multimedia: It is already in the hands of the students to make videos of sports events with their mobile phones. A mobile phone is also an experimental tool measuring for example accelerations. Apps are available showing graphical presentations of the measurements and even allowing an analysis of the data.

In the following, I want to present different **examples** of the combination of sport and physics. In particular, I will cover three topics: **High jump**, what is the **optimal angle** for throwing, and **records**

2. High jump

The question how the height of a jump is determined will lead to different answers – dependent on whom you are asking. A child will say: This is how high I am reaching with my hands. A physicist has the center of mass in his mind and thinks on the energy needed to raise this center of mass. And in sports, the athlete wants to jump over a bar.

Let´s combine first **child and physicist**. This leads to a nice example to measure the force of legs even in a classroom.

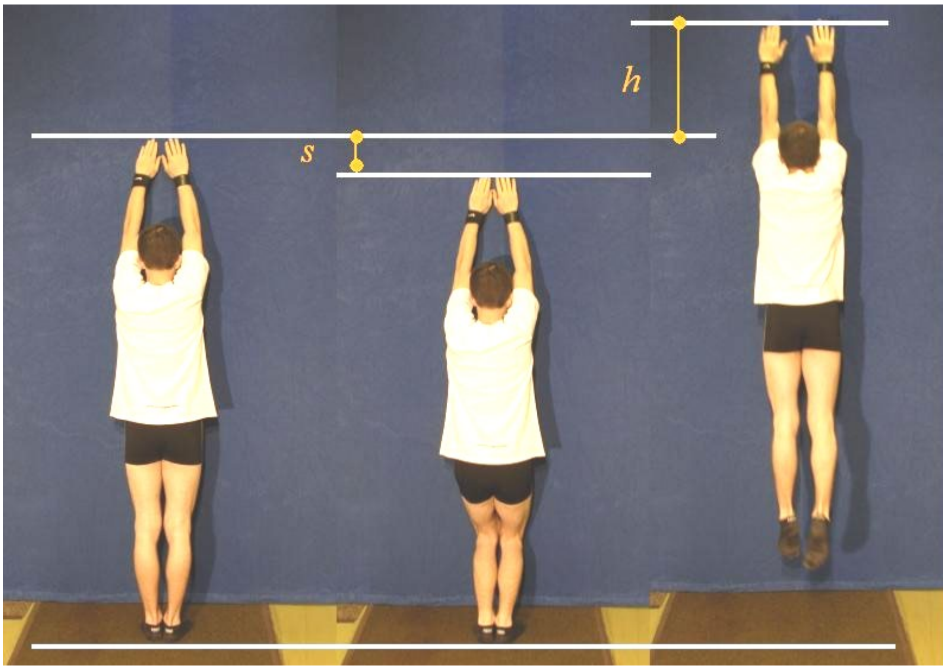

Fig. 2.1: Determination of the force of the legs by high jumping. (From L. Mathelitsch, S. Thaller, Sport und Physik, Aulis Verlag, Köln, 2008)

A student with mass *m* stands towards a wall and makes a mark at the tips of the fingers (Fig. 2.1). Then he bends the knee, makes again a mark and jumps as high as possible to make another mark. Having these three numbers, in particular the two differences, the bending depth *s* and the jumping height *h*, the determination of the force of the legs is straightforward. The force F_L is applied along the path *s*, and the energy goes in lifting the body from the lowest to the highest position, i.e. along the distance *s+h*. This gives the expression for the force of the legs:

$$F_L = \frac{s+h}{s} \cdot m \cdot g \qquad (2.1)$$

Performing this experiment, the students will see that it is not such easy, in particular the determination of the parameter *s*. How deep should one bend the knees? If it is not deep enough or too deep, the jump will be lower. Therefor the students have to find out the optimal bending position first.

Now let´s combine **physics and sports**. Fig. 2.2 show the historical development of different techniques of high jump. From a physics view, it is a decrease of the difference between the center of mass and the bar. This number is given in the last line of Fig. 2.2. One can see that it is even negative for the Fosbury Flop. How can this be? The center of mass can be outside of the body and by a coordinated movement of the upper and lower part of the body, the center of mass can move below the bar, while the body of the athlete never touches the bar.

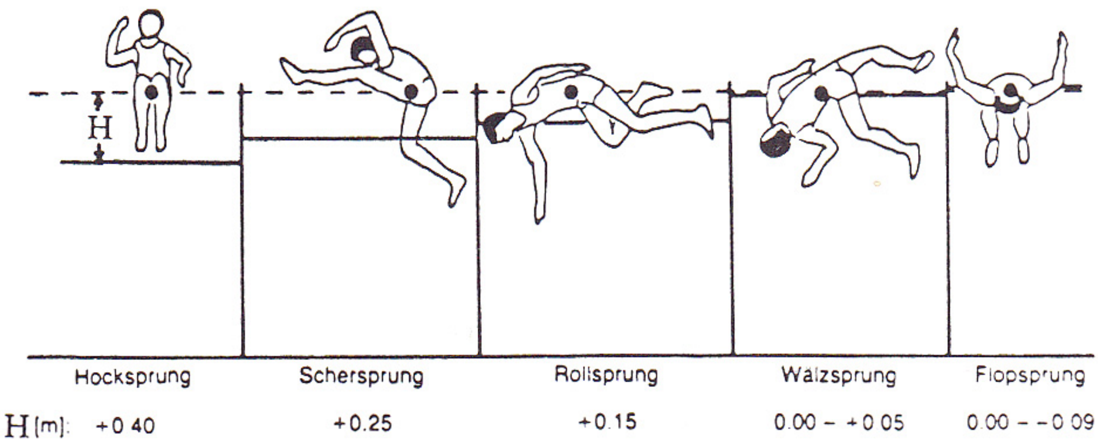

Fig 2.2: Different techniques of high-jump. The last line gives the distances between center of mass of the body and the bar. (From K. Willimczig, Biomechanik der Sportarten, Rowohlt, Hamburg, 1989)

Now the question can arise whether there could be a more profitable technique with which this difference is even more negative. It would be with the belly down since this way of bending is much easier and the center of mass could be farer out of the body. Athletes do not apply this technique. For jumping high, it also needs speed achieved by the in-run. With the Fosbury Flop a smooth transition from running to jumping can be executed. This would be less efficient with the belly-down technique. Nevertheless, there is one sport where this technique is applied: pole vault. Here the run and the jump are disconnected and the athlete jumps belly-down.

Now let´s consider a jump on the moon. How high does one **jump on the moon**? I will try to answer this question with three different models.

Equal velocity means that the jump is done on the earth and on the moon with equal initial velocity *v*. Conservation of energy, or with other words, conversion of kinetic energy to potential energy

$$\frac{m \cdot v^2}{2} = m \cdot g \cdot h \tag{2.2}$$

leads to the formula

$$h = \frac{v^2}{2 \cdot g}. \tag{2.3}$$

Knowing that the gravitational constant on earth is six times of that on the moon gives the result that one jumps on the moon six time as high as on earth. This result can be found in many physics text books.

Equal Force. Why should the initial speed be the same? Let´s start with the assumption that the force of the legs does not change during the ride to the moon. Let F_L be the force of the legs. Then the driving force F_1 for the acceleration a is given by the equation

$$m \cdot a = F - m \cdot g \ = F_1 \tag{2.4}$$

Assuming that F_L = 2 m.g (one can carry a second person on the shoulders), the driving force is different on earth and on the moon

$$F_1^{Earth} = m \cdot g \qquad F_1^{Moon} = \tfrac{11}{6} \cdot m \cdot g \tag{2.5}$$

This yields the result that the velocities before jumping off are different on earth and on moon:

$$v^{Moon} = \sqrt{\frac{11}{6}} \cdot v^{Earth} \tag{2.6}$$

Inserting this relation in equation (2.3) gives a very different results: One jumps on the moon eleven time higher than on earth. So, which model is more correct?

Dynamic model. One has to ask the experts, scientists who do dynamic modelling of the body. Important components of biological movement are muscles. Muscles do not work like a string or a rubber band. They do not obey a law similar to Hookes law, but a very different one:

$$f = \frac{c}{v+b} - a \ . \tag{2.7}$$

The force of a muscle f is inversely proportional to its speed v (a, b and c are constants, dependent on the muscle and on the individual person). The slower the muscle works, the more force the muscle can exert! Fig 2.3 shows the relation which is known as Hill curve.

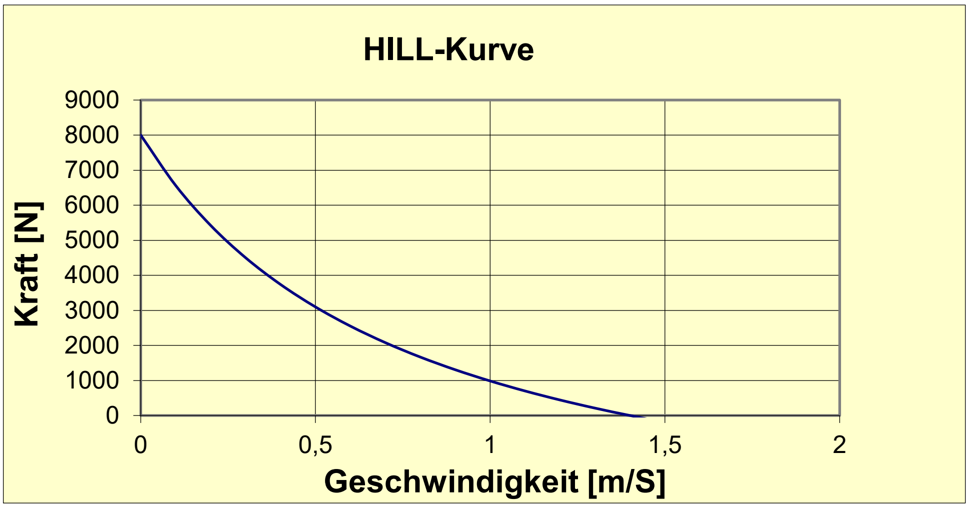

Fig. 2.3: Relation of speed and force of a muscle, called Hill curve. (From L. Mathelitsch, S. Thaller, Sport und Physik, Aulis Verlag, Köln, 2008)

Inserting formula (2.7) into the equation of motion, in addition with expressions for the activation of muscles and the geometry of the body gives the following final result: A jump on the moon should be 10,5 time as high as on earth. That means the second model was by far better.

Why were the jumps of the few persons on the moon not such high? The astronauts were stuck in their space suits. They could barely move and bend their knees. And they were afraid of falling and damaging their space suits.

3. Optimal Angle

There are several sports disciplines where some pieces of sport equipment, like a ball or a discus, are thrown. Sometimes as far as possible, sometimes to hit a target.

In order to throw a ball as far as possible, physicists know that one has to use an initial angle of 45 degrees. The range is given by the following formula, with v the initial speed and α the initial angle.

$$W = \frac{v^2}{g} \cdot \sin(2\alpha) \tag{3.1}$$

In order to maximize W, α has to be 45 degrees. Interestingly, this angle is not applied in any sports discipline. The following examples show that there are different reasons for this.

American Football. In a kickoff or in the transition from offense to defense the ball is kicked as far as possible. But there is another requirement. The own team members run in the same direction and want to come as far as possible. This is dependent how long the ball is in the air. The corresponding time T is given by the expression:

$$T = \frac{2 \cdot v}{g} \cdot \sin\alpha \qquad (3.2)$$

But *T* is maximal for an angle of 90 degrees, straight in the air. So, one has to find a compromise. The relation between *T* and *W* is not symmetric (Fig. 3.1). In the last part, there is a fast fall-off of the range with not too much increase in time. Therefor a compromise would be about 60 degrees and this is the angle with which the kickers try to shoot the ball.

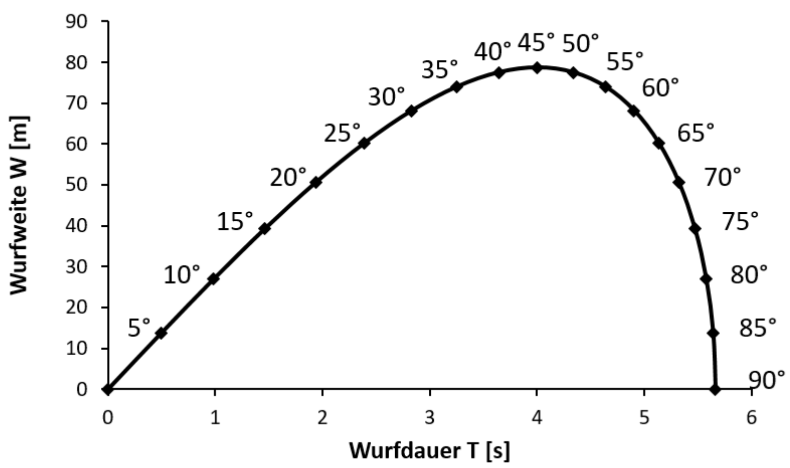

Fig. 3.1: Relation of range W and time T of a football kick. The numbers along the line indicate the angle of throwing. (From L. Mathelitsch, S. Thaller, Der Ball ist unrund, Physik in unserer Zeit 48/2, 2017, 78)

The kick-off is done from the earth and the ball lands on the earth. Very often, the throw starts from a certain height *H*. Therefor we have to extend our expression for the range of a throw.

$$W = \frac{v^2}{g} \cdot \cos\alpha \cdot \left(\sin\alpha + \sqrt{\sin^2\alpha + \frac{2 \cdot g \cdot H}{v^2}} \right) \qquad (3.3)$$

The maximal range is given by

$$W_{\max} = \frac{v^2}{g} \cdot \sqrt{1 + \frac{2 \cdot g \cdot H}{v^2}} \qquad (3.4)$$

achieved by the optimal angle

$$\cos(\alpha_{\max}) = \frac{g \cdot H}{v^2 + g \cdot H}. \qquad (3.5)$$

This relation is shown in figure (3.2).

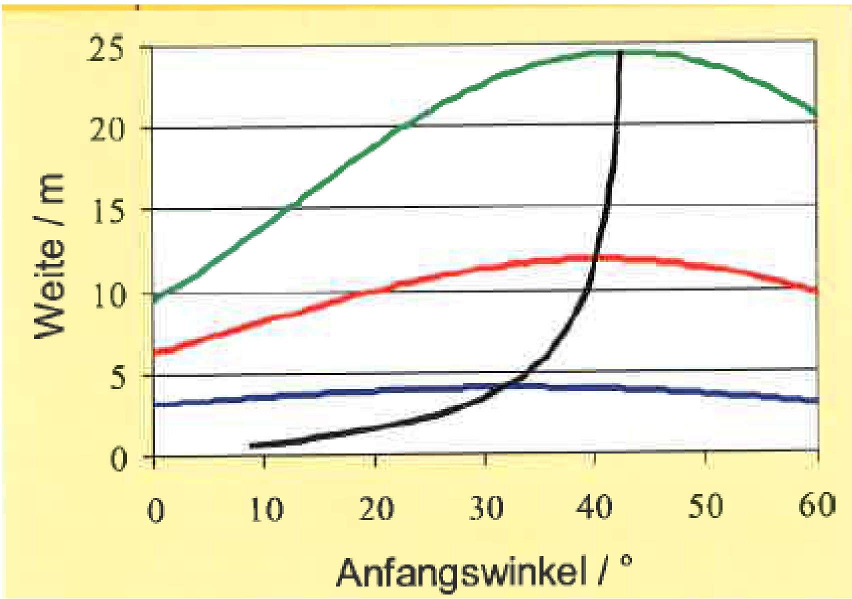

Fig. 3.2: Relation of the initial angle α (horizontal axis) with the achieved range *W* with an initial height *H* of 2 m. The blue line corresponds to an initial velocity of *v* = 5 m/s, the red one to *v* = 10 m/s and the green one to *v* = 15 m/s. The black line connects the maximal ranges. (From S. Thaller, L. Mathelitsch, Steiler oder flacher, Physik in unserer Zeit 42/1, 2011, 40)

Closest to an optimal angle comes the discipline **Hammer Throw**. The hammer is thrown at an angle of 44 degrees, due to its high speed of 25 – 30 m/s.

Shot-put has a lower speed of 14-15 m/s which would result in an optimal angle of 42 degrees. But the geometry of the body forces an angle of around 35 degrees. The athletes have to make a compromise and the actual initial angles are between 38 and 42 degrees.

Long Jump is throwing the own body. The speed of the athlete is fast, almost as fast as in a 100 m sprint. What is the optimal angle? If we assume an initial speed of 9 m/s with an initial angle of 45 degree, the range would be 8,3 m. This is very realistic. But the maximal height of such a jump would be 2,1 m. This is close to a high jump, very unrealistic. What is wrong? To achieve an initial angle of 45 degree, the horizontal and the vertical velocities have to be equal. That means that the athlete would also need an initial speed of 9 m/s in the vertical direction, which is impossible. What is possible is a speed of 3 m/s, at the expense of the horizontal velocity which goes down to 8,5 m/s. This yields an initial angle of around 20 degree. The height of the jump with about 1 m is realistic, but the range of 5,3 m is too low. How do the athletes manage to jump 8 meter?

One important effect is the following: The jumpers raise their arms in the middle of the jump and they put them as low as possible at the end. These shifts the center of mass first higher in the body and then lower. But the trajectory of the jump refers to the center of mass. If the center of mass is lower in the body at the end, it implies that the body is higher with respect to the center of mass. Therefor the body lands quite a bit farther out.

Also with **golf,** experience shows an optimal angle of 20 degrees, but because of a very different reason. In golf, air resistance plays an important role. This is caused by turbulences behind the ball (Fig. 3.3 a). Interestingly, if the surface is not smooth, but has some dimples, then the area of these turbulences is smaller causing less air resistance (Fig. 3.3b). The reason are very small turbulences around these dimples, with the result that the air stream stays longer around the ball

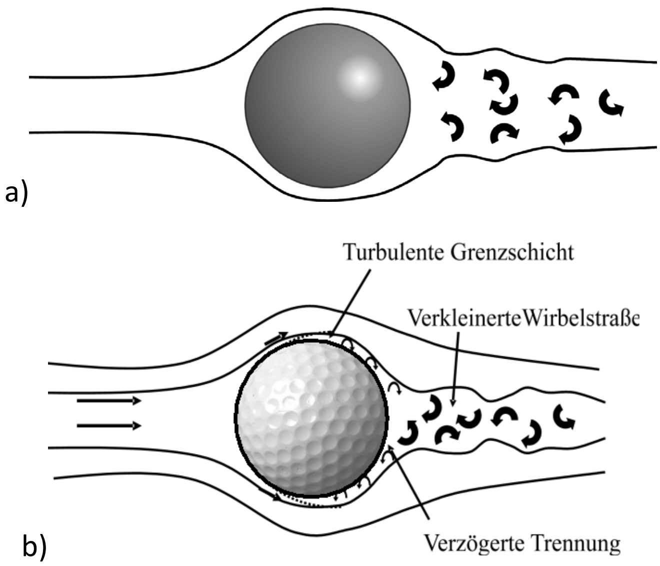

Fig. 3.3: a) Turbulences with a smooth ball. b) Less turbulences with a ball with dimples. (From L. Mathelitsch, S. Thaller, Tückisches Einlochen, Physik in unserer Zeit 40/5, 2009, 252)

If the ball rotates, and it does it very fast, up to 3000 rotations per minute, this changes the direction of the ball (Fig. 3.4). The air stream behind the ball has, depending on the direction of the rotation, a direction to the ground or upwards. This behavior is called Magnus effect and it makes a slice ball flying longer, a top spin ball shorter.

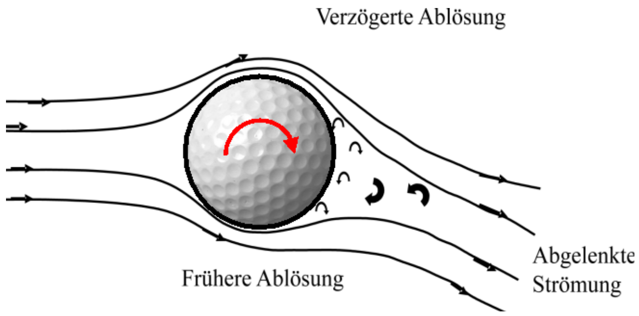

Fig. 3.4: Turbulences behind a spinning ball. (From L. Mathelitsch, S. Thaller, Tückisches Einlochen, Physik in unserer Zeit 40/5, 2009, 252)

This effect has been investigated already in 1910 by J. J. Thompson. This scientist is well known to the physics-community, he is the "father" of the electron. In a discourse "The Dynamics of a Golf Ball" delivered to the Royal Institution, he stated the following: „It is the spin which accounts for the behaviour of a sliced ball.....a golf ball knowing only one rule...always following its nose."

A ball with 60 m/s and at an initial angle of 45 degrees would fly more than 300 m without air resistance. With air resistance, the range would be about 150 m. Applying a slice of 60 rotations per second results in a range of more than 200 m, with an initial angle of about 20 degrees.

Basketball. Research in physics education has shown that students have problems with diagrams. That means that we have to train them. In the following approach, students should transfer reality into a diagram. They should watch a video, in this case of a successful throw of a ball in a basket (Fig. 3.5, left).Then the students should draw what they have seen. They should draw an x-t diagram (horizontal movement versus time) and a y-t diagram (vertical movement versus time). Experience has shown that the y-t diagram is much easier for the students to complete than the x-t diagram (Fig. 3.5, right).

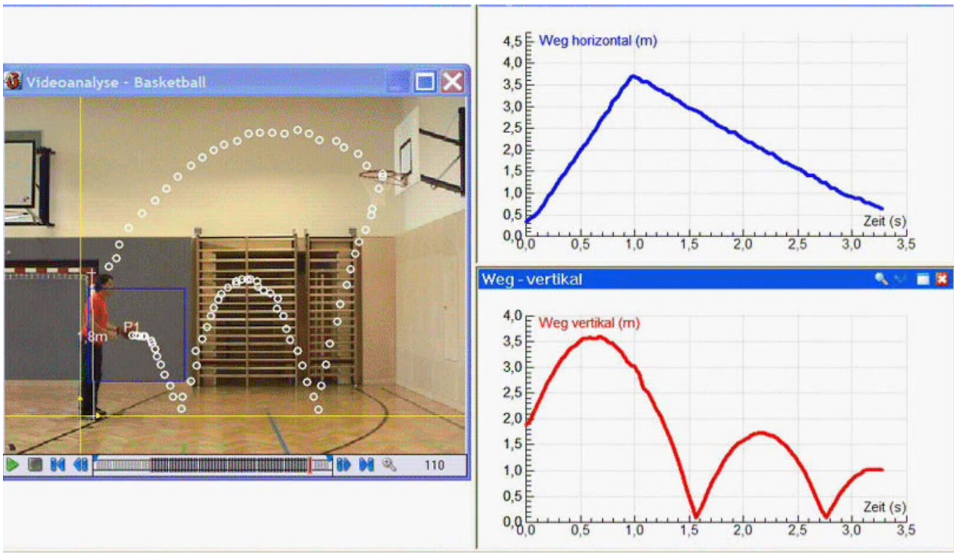

Fig. 3.5: Left: Throw of a ball into a basket. Right top: Horizontal movement of the ball versus time. Right bottom: Vertical movement.

As a second step, the velocity in these two directions can be drawn (Fig. 3.6). Out of experience, here the vertical diagram causes more problems.

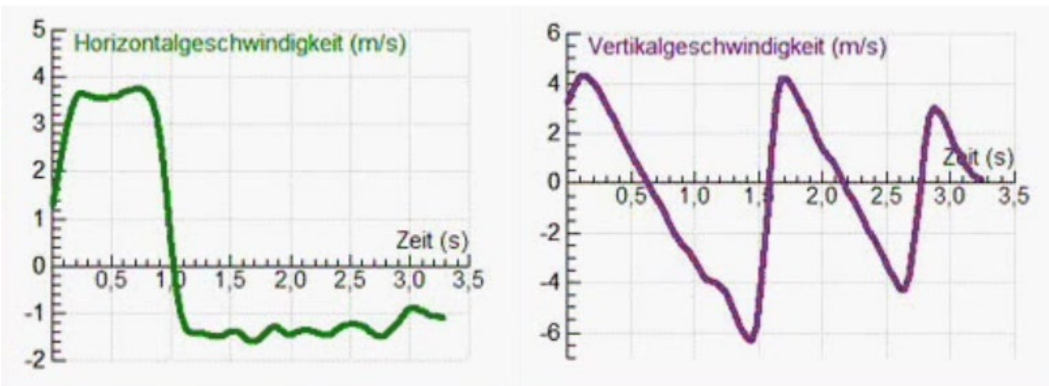

Fig. 3.6: Left: Horizontal speed of the ball of Fig. 3.5. Right: Vertical speed.

And finally the energies, kinetic and potential energy, of the ball can be drawn, as well as the sum of both (Fig. 3.7). As one can see, the ball looses energy more or less only during the contacts, at the basket and at the floor.

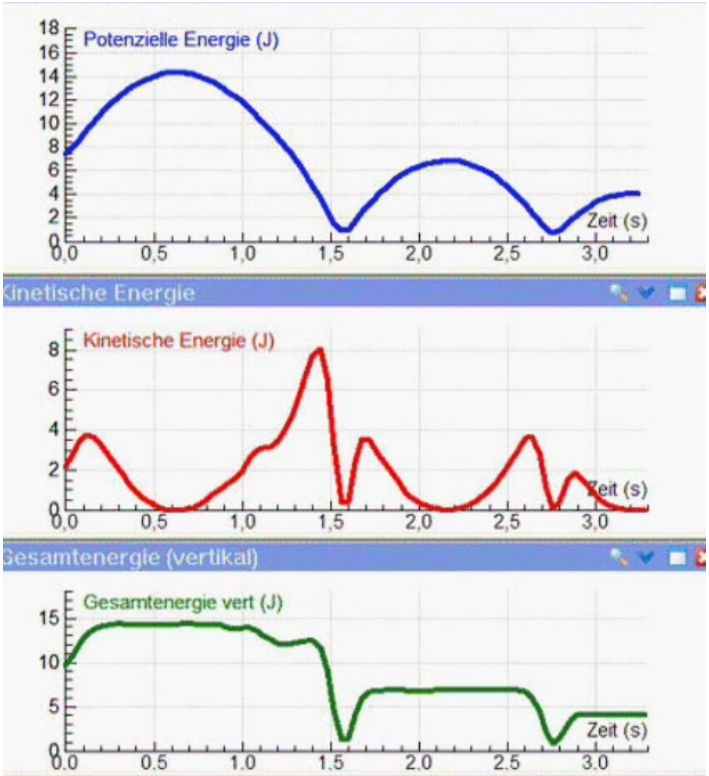

Fig. 3.7: Potential energy of the ball (top), kinetic energy (middle), and sum of both (bottom).

4. Records

To break records is the final goal of many athletes, the own record, that of a country, and most valuable, the world record. There always will be world records. Even if all parameters stay the same, training, equipment, medicine, the attitudes of men and women are not the same, they are distributed, very often with a Gauß distribution. That means that there is always one at the edge with even more positive attitudes than others. And this person can achieve a new record. One just has to wait. Of course, it will become rarer and rarer. On the other hand, there will always be a development – in training, in equipment, in medical support. Therefor in many sport disciplines, one could see a steady improvement in the world records. In this regard, an interesting question arises, namely whether there is an ultimate limit. Of course, a person will never jump as high as 5 meter. But which height could be possible?

I will illustrate this question with one example, **100 m sprint of men**. Fig. 4.1 shows the development of the world record.

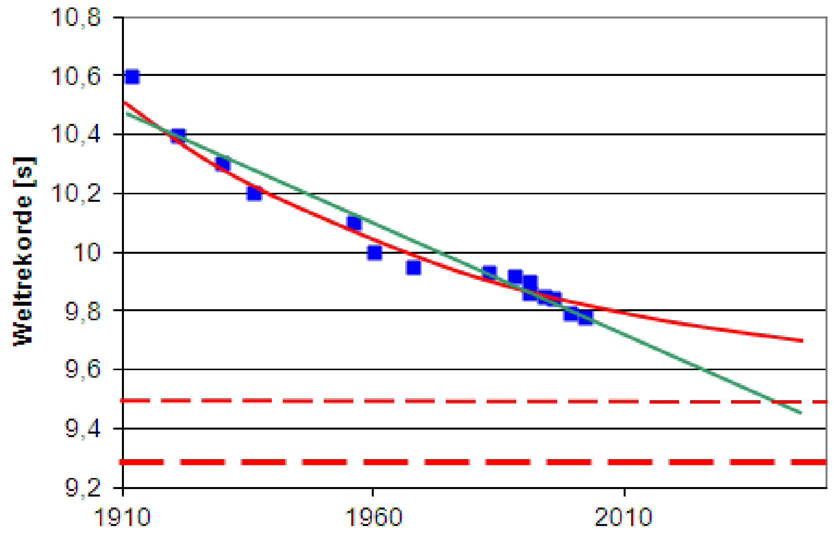

Fig. 4,1: World record of 100 m sprint men. The green and red lines are linear and logistic extrapolations. The dashed lines are estimations of the ultimate limit, see the text. (From L. Mathelitsch, S. Thaller, Physik des Sports, Wiley/VCH, Weinheim, 2015)

If one asks about the ultimate limit, one can try an extrapolation. A linear one (green line in Fig. 4.1) does not make sense since it would end at a time of zero seconds. The red one seems more reasonable, it is an interpolation with a logistic equation. It flattens with a limit of 9,5 seconds (short dashed line). Mathematicians have indicated that world records are rare events. Therefor one has to apply a special statistics, like that for hurricanes or earth quakes. Within such a calculation, one ends with a limit of 9,3 s (long dashed line in Fig. 4.1).

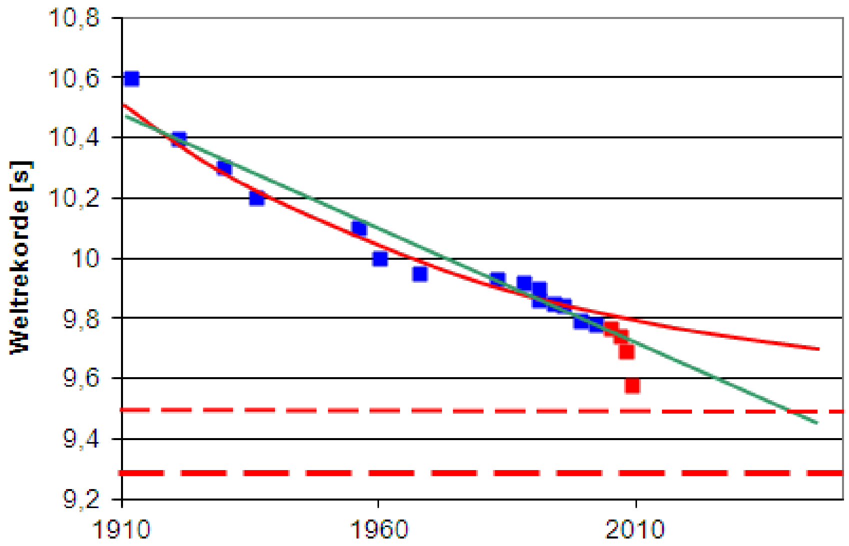

Fig. 4.2: Fig 4.1 with the inclusion of the world records of Ursain Bolt.

Fig. 4.1 did not take into account Ursain Bolt. Including his records, the graph looks different (Fig. 4.2). By now, several years are gone and no new record is in sight. That means the next point on the graph is already quite right and if there will be no new record within the next years, even further out. Then it could be some continuation of the blue dots, of the long-lasting trend from before. And Ursain Bolt was some kind of accident.

5. Literature

In this section, books in English language are listed dealing with the combination of sport and physics.

Adair R. K.: The Physics of Baseball. New York: Harper Perennial (2002), ISBN 978-0060084363

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Goff J. E.: Gold Medal Physics: The Science of Sports. Baltimore: Johns Hopkins Univ. Press (2010), ISBN: 978-0-8018-9322-9