Other Ideas From Malá Hraštice: The Pit and The Pendulum

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The pendulum has been a useful tool in physics since at least the time of Galileo Galilei. The classic "school" formula is said to be valid up to a swing amplitude of 5 degrees. We will see that we can examine larger amplitudes even with simple tools and equipment. The paper adds several ideas for further experiments related to the pendulum.

Introduction

The topics of focus of the spring workshop camp for future physics teachers, organized by MFF UK, has been communicated by several contributions of Physics Teachers Invention Fair, see e.g. [1]. The main topic of this year's workshop camp was "Through Time and Timelessness". The pendulum is related to time; therefore, an opportunity arose to find out how it is with the oscillations of the pendulum, for example with a large amplitude.

The title of the article, as is probably evident, is inspired by the well-known short story by Edgar Alan Poe [2]. (When I thought of the name, I couldn't resist.) To find a physical relationship of the pendulum with a pit was a bit of a challenge, provided that I did not want to talk about the potential well, but even that eventually ended well. But I would like to point out in advance that this part is "a less serious bonus at the end"; the main part of the contribution concentrates on the period of pendulum oscillation.

Pendulum: A Bit of Theory

The formula for the period of oscillation of mathematical pendulum with a small amplitude is given in secondary-school textbooks as $T_0 = 2\pi\sqrt{\frac{l}{g}}$. It is emphasized that the period does not depend on the amplitude of pendulum oscillation. What is the relation for larger amplitudes? The oscillation then ceases to be harmonic and the period increases with increasing amplitude, but non-linearly. The result can be found in university textbooks (e.g. in [2] on pages 411 to 414): $T = T_0 \frac{2}{\pi} K\left(sin\frac{\alpha_{max}}{2}\right)$, where $\alpha_{\max}$ is the maximum amplitude of the pendulum and *K(k)* is a complete elliptic integral of the first kind. (When looking at this statement, a quote from Sherlock Holmes inevitably comes to one's mind: "It's elementary, my dear Watson!")

We see that the relevant theory seems "a little beyond" the level of primary and secondary schools, so it is impossible to teach it at mentioned schools. The possibility that a university-educated physics teacher could at least use it as a wow factor in his or her class (speaking frankly: use it for flaunting) is also probably not very realistic. First, a teacher should not flaunt in front of the students, secondly, majority of students would probably stay completely cool; and finally – a small percentage of students who would be intrigued by the appropriate formula can find the formula on the WolframAlpha website [4]. ☺

(Note for those who would like to try it: Be careful, in WolframAlpha use this relation: $T = T_0 \frac{2}{\pi} K\left(sin^2 \frac{\alpha_{max}}{2}\right)$. The notation of elliptic integral is ambiguous. The same symbol *K()* is used to mark two different functions in different sources). Formally, we could write $K_{Brdlička,Hladík}(k) = K_{WolframAlpha}(k^2)$.)

Pendulum: larger amplitudes in practice

At deflection of 60°, the period of the pendulum compared to small amplitudes is only by about 7% longer; when the deflection is 90°, then it is by about 18% longer. In order for the extension of the period to be significant enough, we apparently need deviations of over ninety degrees. Therefore, a weight suspended on a string won't do the trick. It would be problematic to realize a rigid hinge with a negligible weight, so it is reasonable to use a physical pendulum. A simple construction made at the workshop camp is shown in Fig. 1.

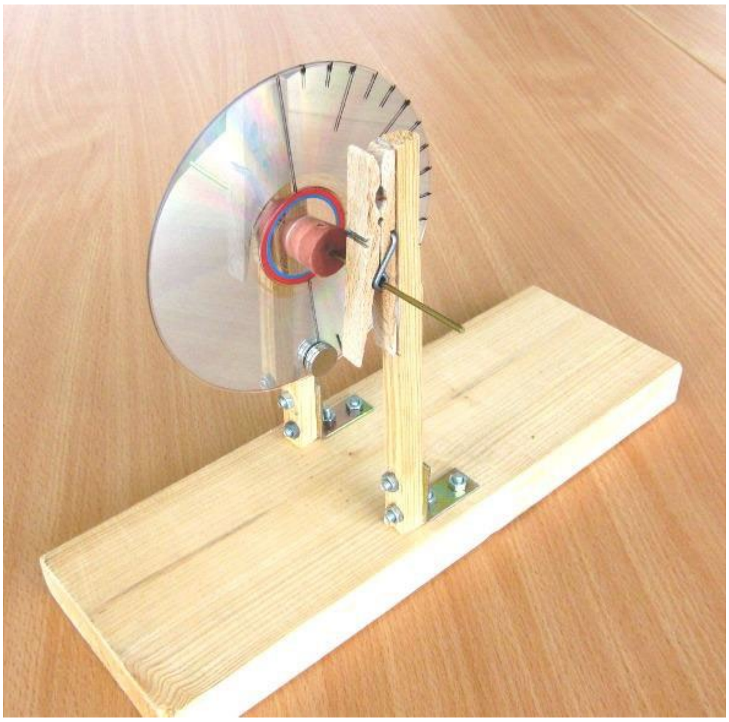

Fig. 1. Simple pendulum design with large oscillations

The pendulum is a CD struck on a rubber stopper of suitable diameter. A hole is drilled through the stopper and a thin brass rod is inserted in it, which serves as an axis. As "bearings" we used wooden washing pegs – the axis rotates in their springs. Two small neodymium magnets are "snapped" on the edge of the CD. By moving them, you can change the distance of the centre of gravity of the pendulum from the axis (and of course the moment of inertia).

Unfortunately, the friction of the axle in the bearings is not negligible; the pendulum makes only about thirty oscillations from the maximum deflection. Lubricating the "bearings" with sewing machine oil brought virtually no improvement. The amplitude of the oscillations therefore decreases rapidly, yet our design can be used at least for approximate measurements.

The detection and analysis of oscillations

The measurement of the period using a stopwatch is highly inaccurate because the oscillation period is only about 0.85 s. Since there are moving magnets on the pendulum, we can use a small coil (just a few turns of bell wire) and the voltage induced by moving magnet can be lead into the microphone input of a computer. The resulting signal can then be uploaded to the Audacity program and we can measure the times when the magnet passed the coil. The time measurement is pretty accurate, but we are missing the information about the amplitude of the oscillation. Therefore, the pendulum movement was filmed and the video analysed.

At the workshop camp, the video was shot in the most primitive way: using a tablet held in hands and leaning against a table. The resulting record was analysed in the freely available program *VLC Media Player* [5] supplemented by the plugin *Jump to time* [6]. In this SW, we can step frame by frame and measure the exact time of the frame.

In the first analysis of the recorded video, the times in which the pendulum has maximum deflections were measured. (It is usually more accurate to measure the times the pendulum passes through the bottom dead centre, but at a frame rate of 30 fps, the pendulum image is blurred in this stage.)

What to compare the measured results with

The theoretical calculation of the oscillation period of a pendulum with large oscillations can be performed much more easily than mentioned above. Article [7] presents a method by which a period can be calculated, for example in Excel. We will not give a justification here, only the algorithm itself: We denote $a_0 = 1,\ b_0 = \cos(\alpha_{max}/2)$ and then we determine the arithmetic and geometric mean: $a_1 = (a_0 + b_0)/2,\ b_1 = \sqrt{a_0 \cdot b_0}$. We proceed in the same way in the next steps. $a_2 = (a_1 + b_1)/2$, $b_2 = \sqrt{a_1 \cdot b_1}$ and so on. The quantities *a* and *b* converge very quickly to each other; after 4 iterations they differ by less than 10^{-8} for $\alpha_{max} < 170°$. If we denote the limit by a_∞, the pendulum period is $T = T_0/a_\infty$.

Comparing the experimental results with theory

A comparison of the measured results with the theoretical prediction is shown in Fig. 2.

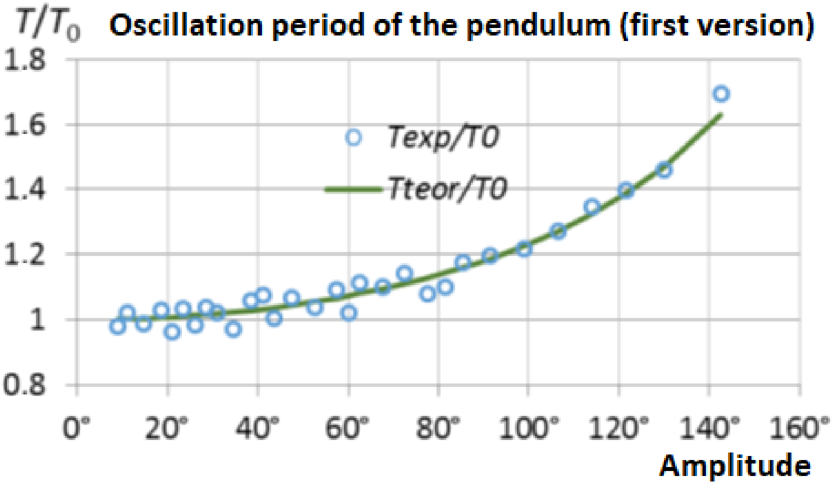

Fig. 2. The oscillation period of the first version of the pendulum vs. its amplitude

It can be seen that despite considerable simplicity of the pendulum construction, the measurements provide results that are surprisingly in accordance with the theory. Especially with smaller pendulum deflections, however, the measurement errors are estimated at 5% or more.

Nevertheless, for more accurate and conclusive measurements, it is necessary to improve the design, especially to reduce friction.

Improved versions of the pendulum

Subsequently, two improved versions of the pendulum were created. One is still very amateurish; we need only a drill for its construction. A CD is again pressed onto a rubber stopper, see Fig. 3. A thick needle for sewing leather is used as an axis and small beads as bearings. The friction in the axis is somewhat smaller in this construction, the pendulum oscillates more than fifty times until it stops.

Another improved construction was developed by Ing. Ludvík Němec from KDF MFF UK, see Fig. 4. The axes taken from model trains rotates in tip bearings (also from models, both parts were purchased as spare parts). In this construction, the friction is considerably smaller and the pendulum stops after four hundred oscillations.

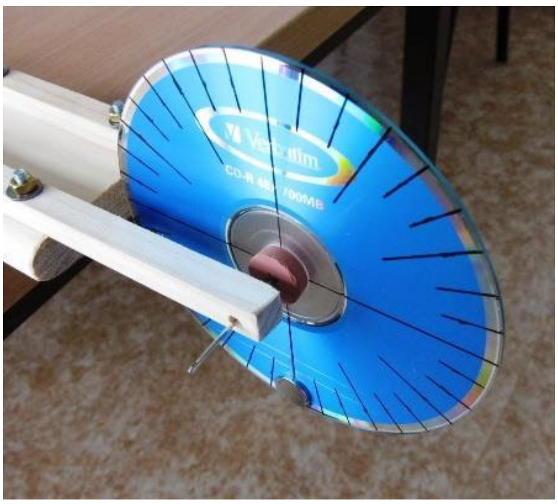

Fig. 3. Improved version of the pendulum

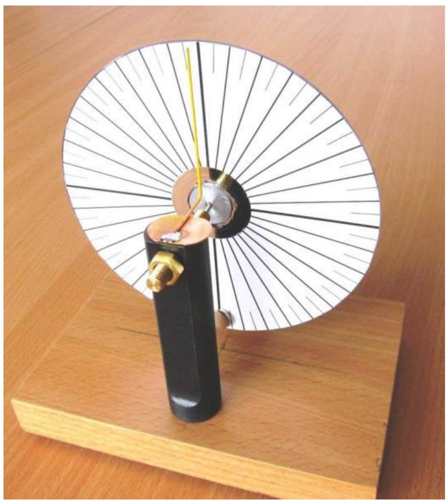

Fig. 4. "Professional" version

The motion of the two pendulums was recorded using high-speed video camera, Casio Exilim EX-FH 20 at the frame rate of 210 frames/s, and subsequently analysed again in the VLC Media Player software. The passage times through the "bottom dead centre" were measured. The accuracy of time measurement can be estimated at about 3 ms, the accuracy of amplitude reading at about 1°.

The results for the best ("professional") version of the pendulum are shown in Fig. 5; the measured results usually do not deviate from the theoretical by more than half a percent, the maximum deviation was about 0.8%. Even the measurement using the pendulum from figure 3, however, gives good results, the deviation of the measured periods compared to the theoretical values does not exceed 2% and usually is in tenths of a percent.

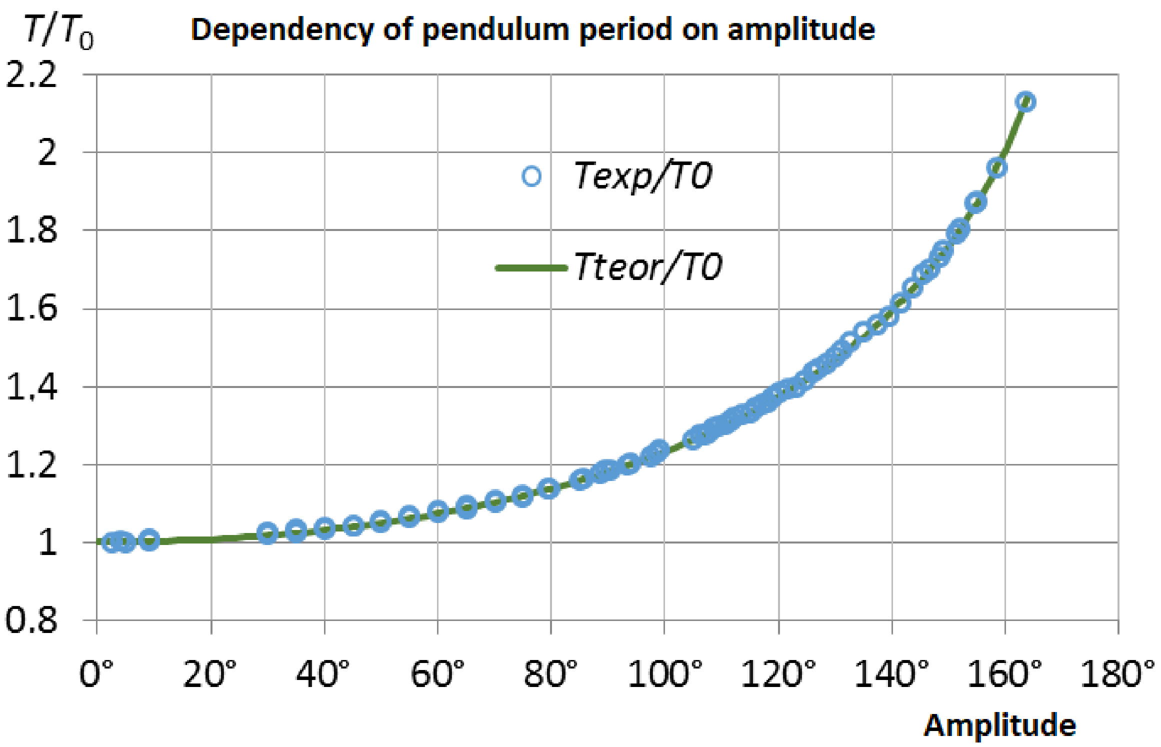

Fig. 5. Comparison of measured and theoretical periods of pendulum oscillations (for the pendulum version as shown in Fig. 4)

The fact that pendulum period lengthens at larger amplitudes and that it corresponds to the theory can therefore be illustrated and measured with sufficient accuracy; at the same time, simpler constructions could be made by the pupils themselves, for example within the framework of various projects. Another possibility of using the created pendulums is to show the dependence of the period on the moment of inertia (by adding other magnets to the CD) or the dependence of periods on the gravitational acceleration – when tilting the pendulum axis (so-called Mach's pendulum).

The Pit and the Pendulum

And finally, something concerning the possible physical connection of the pit and the pendulum. We could even say with a little exaggeration, "the pit *is* the pendulum." If we let a ball roll back and forth in a spherical pit, it performs harmonic oscillations (at small oscillations). From the theory it is possible to deduce that the oscillation period is

$T_0 = 2\pi\sqrt{(7/5)(R/g)}$, where R is the radius of the spherical surface (or rather more precisely, the difference between the radius of the spherical surface and the radius of the ball). We can use this to measure (rather unusually) the radius of curvature of the hollow mirror and compare it with the result that comes out of the optical measurement. For a cheap cosmetic mirror (in which there was a problem to determine its focus point), the results matched better than to 10%.

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