Other ideas from Malá Hraštice 4: the gravity of Earth in one hundred different ways

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The contribution presents several suggestions for experiments which have been tried and newly designed at the spring training camp for future and present physics teachers in Malá Hraštice. It is not about the most accurate measurement, but about how students can practise various parts of mechanics from primary school level to high school level on the task to measure gravity. Some methods of measurement are a bit unconventional or even peculiar, so you can choose them if some nosy students are bored with "experiments from the textbook".

Introduction

Spring training camp for physics teachers in Malá Hraštice had been an inspiration for some contributions in the previous years of the Physics Teachers' Inventions Fair, see, for example, [1, 2]. In 2014, "mini projects" at that camp were dedicated to mechanics. In experimenting, I found interesting the following question:

In what different ways could we measure gravity?

Some ideas, which were partially elaborated after the camp, are presented in this contribution. "In one hundred different ways" in the title is an exaggeration, of course. I would not feel up to inventing of one hundred methods, even if they were overly complicated. One hundred is about fifteen times as much as the number of methods that will be presented here, but if you repeat the measurement many times (and calculate the results to a lot of significant figures), you could definitely obtain even two hundreds of different values of g… ☺

As it was mentioned above, we do not want the most accurate result of the following measurements, but we mainly want to play with physics.

The classic method of determining the gravity is using the mathematical pendulum, in case of school conditions, for example, a nut hung on a thread. It gives rather accurate results, but requires the knowledge of the relation for the oscillation period. But could it be easier?

1. Free fall (from the "free hand ")

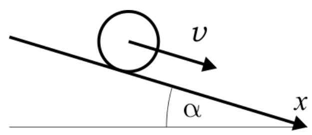
Let us try just to let a small object fall from the hand lifted straight overhead and estimate for how long it is falling. If we count "Missisippily", we will be able to estimate that the time of falling is between one half of a second and one second. Wellknown equation for the height of free fall stands that $s = \frac{1}{2} g t^2$, which implies that $g = \frac{2s}{t^2}$. The height of free fall is around 2 m; for the time of falling equal to 1 s, we would obtain the value g = 4 m/s^2, and for t = 0.5 s we would obtain 16 m/s^2. It certainly is a very rough estimate, but on the other hand, in case of some measurements we are glad if the result is correct within an order of magnitude, and error of 60 % does not bother us ☺. However, this "measurement" should be presented to students as a really rough estimate, and should not been improved by taking an average of 4 and 16 with the following rejoicing at the resultant error of 2 %. (I am afraid that not all of students would get the joke, only truly good ones maybe.)

Could this rough measurement be made more accurate? Using a stop-watch, we can measure the time of falling more precisely. For instance, measured values of the time during which tennis ball is falling from the height of 2.1 m ranges from 0.62 s to 0.70 s, which corresponds to the range of g from 8.6 to 10.9 m/s^2. The maximum error is a little over ten percent, furthermore, the average of more measured values is ca. 0.66 s, from where it follows that g is 9.6 m/s^2, which is very close to the table value.

Rather "psychological" note to the accuracy of the measurement: Since we all are probably used to "the value from the textbook" of 9.81 m/s^2, usually it could seem that if we obtain 9.7 or 9.9 by our measurements, we still do not "measure right" so we should make the measurement more accurate. Although in our case, we are just one percent away from the expected value! What we would give for this when measuring some other quantities. Thus we should not be exaggeratedly severe especially with such simple measurements.

2. Ball on an inclined plane

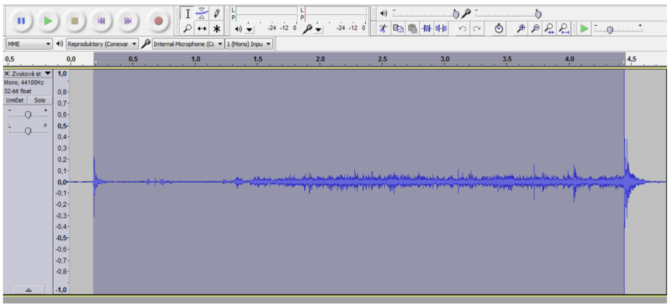
In case of the above mentioned measurement, there is a problem with measuring such a short time of falling by a stop-watch with sufficient accuracy. We can prolong the fall in accordance with Galileo Galilei's model by letting a suitable body, for example a small ball, roll on an inclined plane. Calculating the gravity, however, can be slightly complicated by the fact that the ball does not slide, but rolls with the acceleration which is lesser than the acceleration in case of sliding. It is because its gravitational potential energy transforms not only into the energy of the ball's translational motion but also into the energy of its rotational motion. In case of a homogeneous ball rolling with the velocity *v,* the total kinetic energy is equal to $E_k = \frac{1}{2} \cdot \frac{7}{5} m v^2$. As for calculating the acceleration of the rolling ball, it is easy at the university level (it is a beautiful illustration of Lagrange's equations of the second kind). Computing the same result at the secondary-school level is rather lengthier, but surprisingly it is also possible to derive from the above mentioned relation for the kinetic energy, even though it would probably be more suitable for a seminar for interested students. Unfortunately, there is no space for this derivation, so let us state only the



resultant equation for the covered distance on an inclined plane during the time *t*:

$$s = \frac{1}{2} \cdot \frac{5}{7} \cdot g \cdot \sin\alpha \cdot t^2$$

How to measure time? Either manually by a stop-watch or by a computer and microphone (the ball will tap on some stopper at the end). At the start of the motion, we can also tap on the wooden stick for example, with which we are holding the ball at the start point. We record the sound scanned by the microphone with the Audacity application, in which we can easily measure the time of the ball's rolling (see picture).



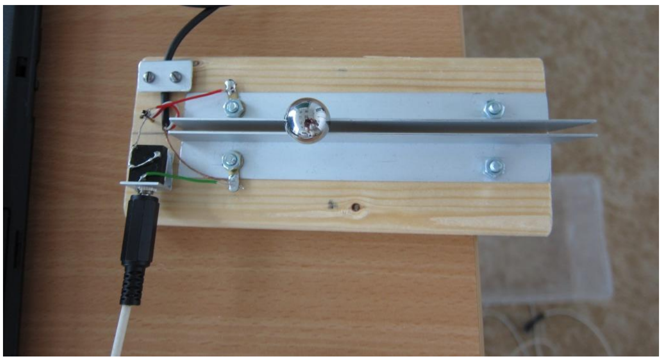
We will use the Audacity application for measuring in another method as well.

3. Free fall of a ball – more accurate measurement

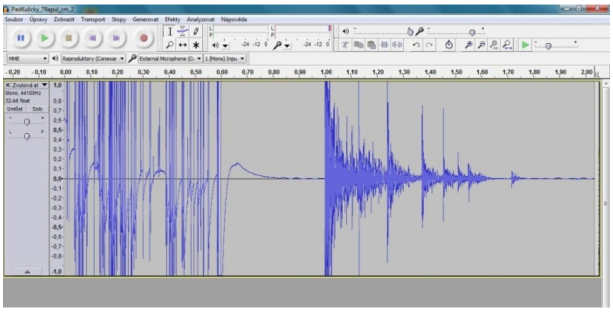
There was a simple idea at the beginning of the following method: We will let the ball roll on a table and fall down on the floor. As long as it rolls on the table top, it will be "noisy". The moment when the ball left the table top is obvious from the sound record (noise of the rolling will stop). We will also hear the ball hitting the floor. Then we will easily measure the time of falling using Audacity.

However, all theory is grey! Sound that we hear when the ball is rolling is actually emitted by the table top and it turned out that the table vibrates even for almost one tenth of a second after the ball is already falling! Since the time of falling from the table which is 75 cm high is about 0.4 s, then supposing the resultant accuracy of 5%, we would need to measure the time of falling with the accuracy of around 20 ms, and we just cannot determine start of falling from the sound with this accuracy.

Nevertheless, we can come up with another way, how to determine the moment of starting falling with the accuracy of even fraction of a millisecond. Principle and possible solution are shown in the photo. Metal ball lies on two small "rails" which, in our case, are made of two aluminium L-shaped sections screwed to a wooden board. Distance between the sections is about 4 mm. Rails are connected in parallel to the microphone which is connected to a computer's microphone input. As long as the ball is lying on the rails, it short-circuits the microphone input, but as soon as it goes off the rails and starts to fall, short circuit will be cleared off and computer will record it as an immediate signal change. Then the microphone will record the sound of the ball's impact.



In the photo there is a plastic container in the place where the ball falls. When the ball hits the bottom of the container, the sound is strong enough, besides we do not have to look for the ball that rolled somewhere, god knows where. It is better to place the microphone near the container, so that we do not have to take into account the time-lag caused by sound propagation; distance of 1 m would cause the time-lag of 3 ms.

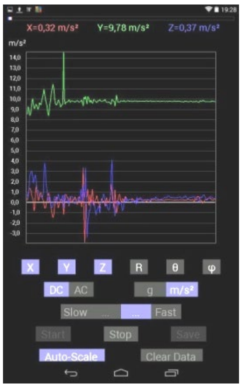


In reality, waveform of the recorded sound is not so "idyllic" as we have described above. Apparently, the ball mildly bounces on the rails or lose the contact in the places where aluminium was oxidised, so there is a lot of interfering impulses in the signal, see the picture at the right. Fortunately, the point where the ball leaves the rails still can be well located. (Let us add two technical notes: 1. Decreasing of the signal and its overshoot after the last impulse is probably given by capacitors in the computer's microphone input. 2. The ball has to have a certain minimum velocity on the rails so that it did not flip over the end edge slowly; calculation will show that for a small ball the velocity of about 40 cm/s is enough.)

Results obtained by this method give *g* really close to 9.8 m/s^2. It is even possible to measure g by letting the ball fall from the rails onto the table, which means the height of three centimetres only!

4. Measurement by tablet or mobile

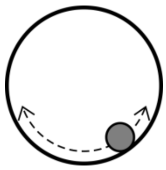
For contemporary tablets and mobiles there exist many applications which measure acceleration. However, some of them, even good ones, indicate values which are multiples of *g*, which will not really help us with the measurement. (Perhaps we can at least calm down about the fact that the value of gravity in our surroundings is more or less standard.)



From the applications which measure acceleration on tablets and mobile phones with Android operating system and which are free to download on Google Play and which will really indicate acceleration in m/s^2, we can use, for example, *Accelerometer* [3] for the simplest measurements, see picture on the left. This application just shows acceleration components and its magnitude with the accuracy of one decimal place. More developed Accelerometer monitor [4] (see other two pictures) indicates either acceleration components, or magnitude and direction of the acceleration, and also the time dependence of these two quantities. We can also choose, whether units will be m/s^2 or g.

But pay attention to the accuracy of the measurement! The fact that the values are displayed with two or three significant figures does not guarantee anything. (In case of my tablet, its rotation of 90 degrees caused the change in the indicated value of the acceleration magnitude from 9.8 to 9.3. Thus, it depends on the quality of our device's sensor and on its calibration.)

5. Ball in a tube as a pendulum



This is another non-traditional option, where computation is more a university problem. Ball rolls within a cylinder and so it performs oscillating motion. If we calculate the period of the cylinder (let us recall the part Small oscillations of the course of theoretical mechanics), we will ascertain, how it depends on g. We can again measure the period by Audacity; we will easily identify maxima and minima of the velocity of the rolling ball from the sound record. Factual measurement (for the ball of the diameter of 1 cm in the tube of the internal diameter of 3.6 cm) has given after calculation the value of g equal to approximately 9.7 m/s^2.

6. Weight on a spring

Weight on a spring is a favourite example for illustration of the oscillating motion. Surprisingly, the gravity acceleration can be also measured by it, and that all without knowing the mass of the weight and stiffness of the spring! It is sufficient to measure the change in the spring's length after hanging the weight (in our case the change was x = 14 cm) and period of oscillations (in our case T = 0.76 s). If we substitute these values into the expression $4\pi^2 x/T^2$, we will obtain the value (in our case 9.6 m/s^2) which gives g determined by this method.

Why is that so? Just combine the relation for the period of oscillations with the relation for lengthening of the weighted spring. Mass of the weight and stiffness of the spring will be reduced and we will get the result. (Let us note, that we do not consider mass of the spring, but if the weight is not extremely light, it will not affect result so much.)

7. Coin in a balloon

You must see this experiment or better try it, otherwise you will not believe. Put a scalloped coin into balloon. Then blow the balloon up, hold it in both your hands and move it along a circle (axis of the motion is horizontal). At first, the coin in the balloon is hopping, but after several attempts you will manage to do so that the coin will start to circulate on the internal side of the balloon. As the coin is scalloped, membrane of the balloon is shivering, when notches strike it, and we hear the tone, which is higher if the coin moves faster. From the tone height we can determine the coin's velocity. When the is coin at the bottom of the balloon, the velocity is higher than in the case, if it was up in the balloon. Difference is given by the different potential energy at the bottom and at the top. And since the difference between the potential energies is proportional to g, we can calculate its value from the frequencies (for instance, measured by Audacity again) and diameter of the balloon and the coin. Measurement goes better when the coin rolls in the balloon slowly. (Relative difference between the frequencies of the sound in the cases, when the coin is up and down, is greater.) Even this very non-traditional measurement gives the values from about 9.5 to 9.6 m/s^2.

Conclusion

I hope, that even without more detailed calculations, which did not fit into this contribution, some of the above mentioned methods can be an inspiration for you to try them with your students. Best of luck with these and other experiments!

References

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