What is the maximum acceleration of a car?

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Introduction

According to the information in the mass media [1], a German company is preparing a sports car, which will be able to increase its velocity from zero to one hundred kilometres per hour in less than one second. A physicist will react immediately: Is it possible at all? What are the physical limits to acceleration of the wheel-driven vehicle? Contemporary sport cars cannot accelerate from 0 to 100 km/h in less than two seconds. Only high powered race cars with special tyres are capable of that (see Table 1).

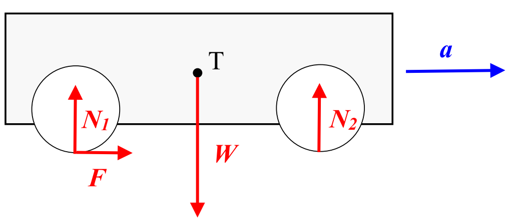
vehicle type	accelerating 0-100 km/h [s]	acceleration $[m/s^{-2}]$
Renault F1	1.7	16.3
Bugatti Veyron	2.46	11.3
Porsche 911 Turbo	2.7	10.3
Ferrari F12 Berlinetta	3.1	9.0

Table 1

Two different models for calculating acceleration limits will be presented in this contribution. The first model, which is frequently represented, is not physically correct; however, it could be a satisfactory approximation. Discussion of the results of the second model practically eliminates the possibility that an axle-driven car could achieve such high acceleration expected by the manufacturer.

Model 1

We often find the following solution approach. Let us assume, that the car has one drive axle (for example, at the rear of the vehicle) and the longitudinal location of the centre of gravity is in the middle between the axles. Effect of the rotational mass is not considered. Model is based on picture 1. In the vertical direction, the weight



Pic. 1

is compensated by the normal forces on the axles, which are equal because of the selected position of the centre of gravity (point T in the picture). The maximum accelerating force is given by the coefficient of sliding friction f and the following formula can be applied to it[1]

$$F_{max} = fN_1 = f\frac{W}{2} = f\frac{mg}{2}, \qquad (1)$$

where m is the mass of the car. Then the maximum acceleration is equal to

$$a_{max,1} = \frac{F_{max}}{m} = f\frac{g}{2}. \qquad (2)$$

A very simple result is obtained. It is curious though, that the assumption that the car is rear wheel drive was not used, even if we know from experience, that rear-wheel drive cars with the same relative axle loads accelerates on the slipper surface better than front-wheel drive ones[2]. Therefore, model no. 1 does not describe the acceleration of the car correctly.

Model 2

Force analysis shown in picture 1 is erroneous. As we know, during the process of accelerating the vehicle only moves translationally, which means it does not rotate. Thus, the resultant moment of force has to be equal to zero and evidently forces in picture 1 do not meet this condition. In order for the forces $\boldsymbol{N_1}$ and $\boldsymbol{N_2}$ to compensate the moment of the force $\boldsymbol{F}$ that accelerates the car (we calculate the moment with respect to the centre of gravity), the magnitude of the force $\boldsymbol{N_1}$ must be greater than the magnitude of the force $\boldsymbol{N_2}$. Proper force analysis is shown in picture 2.

As long as the vehicle does not accelerate in the vertical direction, vertical forces must be in equilibrium

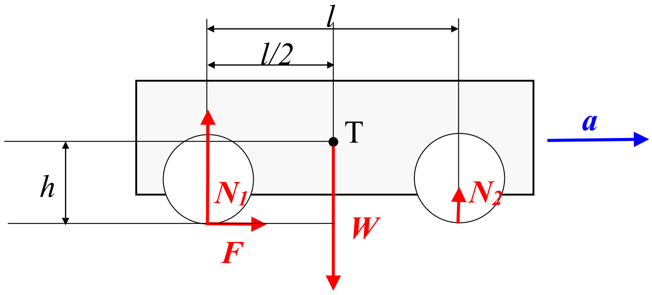
$$W = N_1 + N_2. \qquad (3)$$

From the condition of equilibrium of moments, it follows that

$$F_{max}h + N_2\frac{l}{2} = N_1\frac{l}{2}, \qquad (4)$$

where F_{max} is the maximum accelerating force, for which it holds that

$$F_{max} = fN_1. \qquad (5)$$



Pic. 2

Meaning of other symbols is clear from picture 2. Equations (4), (5) and (6) generate a linear system with three unknowns F_{max}, N_1 a N_2. Solving the system, we obtain

$$a_{max,2} = f\frac{g}{2\left(1-\frac{hf}{l}\right)}. \qquad (6)$$

[1] Symbols for physical quantities that are vectors, such as force, moment of force and acceleration, which are not in the bold type, represents their **magnitudes** or vector **components** (horizontal or vertical). The situation is always obvious from the context or pictures.

[2] Similarly, the front axle is more effective for deceleration, that is why the front brakes are larger than the rear brakes.

On one hand, the result of model 2 has met our expectations that it leads to the higher value of the maximum acceleration than model 1 gave us. During accelerating the rear wheels are more loaded (at the expense of the front wheels), therefore a rear wheel drive vehicle has an advantage. However, on the other hand the result does not seem sensible because as long as the height of the centre of gravity is decreasing and the wheelbase is widening, the maximum acceleration is increasing ad infinitum or can even attain negative values!

Limitation of the model no. 2

Car acceleration could diverge as long as the force N_1 also diverged. Then, however, the force N_2 would have to point downwards, which is not possible for the car. Model no. 2 thus holds true only under the condition that $N_2 > 0$. In case of breaking this condition, the front wheels of the car would be lifted.

We express vertical component of the force N_2 from the equations (4), (5) and (6).

$$N_2 = \frac{mg\left(1 - \frac{2f \cdot h}{l}\right)}{2 - \frac{2f \cdot h}{l}}. \tag{7}$$

Condition $N_2 > 0$ implies inequality

$$\frac{2f \cdot h}{l} < 1. \tag{8}$$

As long as this requirement is satisfied, sliding friction between tyres and road will limit the acceleration. Otherwise, acceleration will be limited by lifting the front wheels up.

Limitation of the model no. 1

Let us compare the results of models no. 1 and 2 (equation (2) and (6)). We see, that figuring equilibrium of moments in the second model cause a correction, which increases with the value of the coefficient of static friction. Provided that f is small enough, model 1 would be able to give results with acceptable accuracy. For illustration, let us calculate, under what conditions model 1 would provide results, whose deviation is less than 10%

$$a_{\max,2} - a_{\max,1} < 0{,}1 \cdot a_{\max,2}. \tag{9}$$

After substituting the accelerations from the equations (2) and (6) and adjusting the relation, we obtain

$$f \cdot \frac{h}{l} < 0{,}1. \tag{10}$$

Therefore, model 1 can be a good approximation for sufficiently small values of the coefficient of sliding friction and low centre of gravity.

Four-wheel drive car

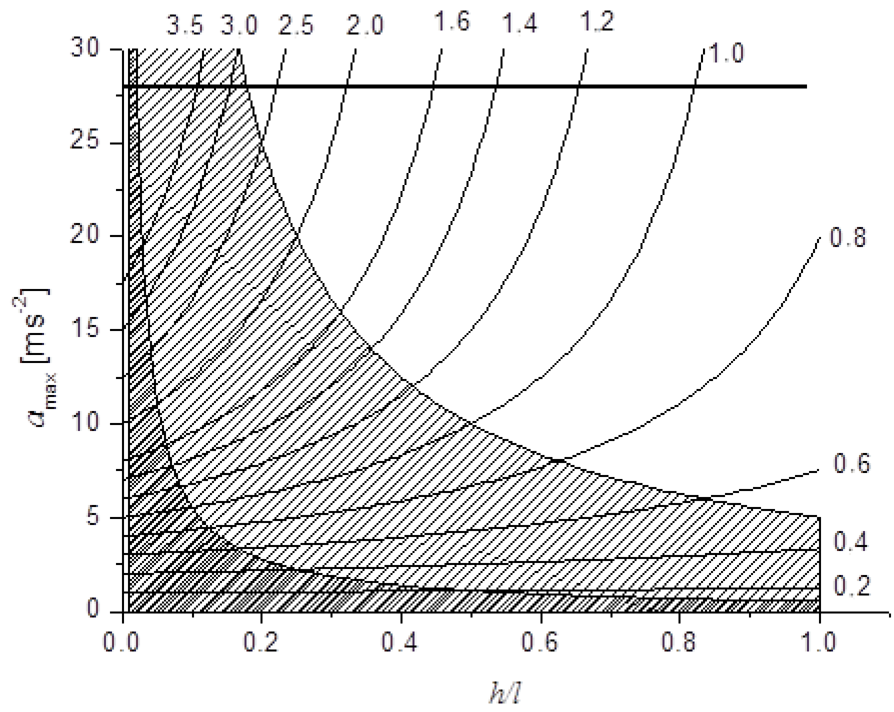
Surprisingly, solving of the maximum acceleration of an all-wheel drive vehicle is easier than in case of a vehicle with one drive axle. During the acceleration though, normal forces redistribute between the front and rear axle, but the magnitude of their sum is always equal to the magnitude of weight. If the car could control traction so that both axles have reached the skid limits, the maximum acceleration would equal

$$a_{\max} = \frac{F_{\max}}{m} = \frac{f \cdot mg}{m} = f \cdot g. \qquad (11)$$

Summary

The results are summarized in pic. 3, in which maximal acceleration of the rear wheel drive car is represented as a function of the fraction *h/l*. Individual curves correspond to the different coefficients of static sliding friction (numbers inside the graph). Realistic situation is depicted by the hatched area; outside of this area the car is going to flip over. Boundary curve of the hatched area also gives the maximum acceleration of the four-wheel drive car, which does not depend on the location of the centre of gravity. Inside the densely-hatched area error of model 1 is less than 10%.

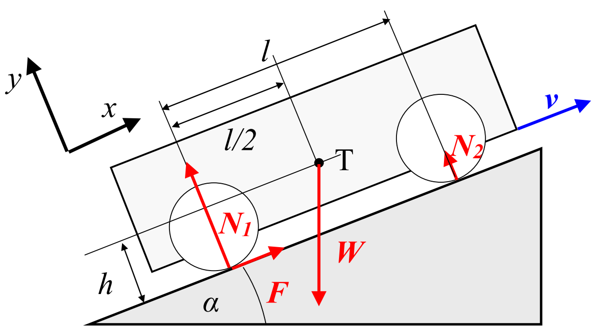
In order for the car to accelerate from rest to 100 km/h during 1 second, its average acceleration would have to be about 28 $m \cdot s^{-2}$. This value is highlighted in the chart with the thick horizontal line. To get this value, the coefficient of sliding friction has to be at least 3 irrespective of the car construction. Common values of the coefficient of sliding friction between tyre and concrete range from 0.7 to 0.8, for asphalt these values are even lower. Higher values can be attained only in case of special tyres or modified surface (drag racing). However, for the higher coefficient of sliding friction we pay with very short service life of tyres. Acceleration from 0 to 100 km/h during 1 s, which the German manufacturer expects from the planned automobile, seems to be unreal.



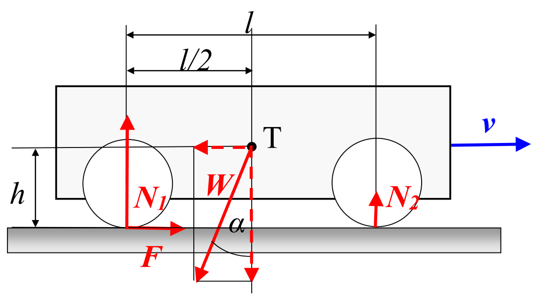
Pic. 3

Possibility of experimental verification in the laboratory

Direct experimental verification of the phenomenon at high acceleration is difficult. Short time of accelerating and long displacement basically eliminate realization of these experiments in school laboratories. However, we can make use of the analogy between the uniformly accelerated horizontal motion and uniform motion on an inclined plane, see pic. 4.



Pic. 4 (a)



Pic. 4 (b)

As we use this analogy, equations (3), (4) and (5) will get following forms

$$mg\cos\alpha = N_1 + N_2, \quad (12) \qquad F_{\max} h + N_2\frac{l}{2} = N_1\frac{l}{2}, \quad (13) \qquad F_{\max} = fN_1, \quad (14)$$

where, furthermore, motor tensile force compensates the component of the weight in the direction of motion, thus

$$F_{\max} = mg \sin \alpha. \tag{15}$$

After solving the system of equations we obtain

$$\sin \varphi = \frac{f}{f + 4\left(1 - \frac{f h}{l}\right)^2}. \tag{16}$$

Problem of acceleration measurement then will be transferred into the experimentally much easier problem of determination of the angle of the inclined plane at which the drive axle slips or the car flips over.

Reference

[1] http://auto.idnes.cz/geniove-nebo-tluchubove-z-nuly-na-sto-pry-pod-sekundu-pda-/ak_aktual.aspx?c=A110827_202252_ak_aktual_ada