Presumptions in physics and life

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Introduction

Evolving the ability to estimate the values of different quantities like mass and length and also the amount, that could be useful in everyday life, should be a part of physics education. The subjects of this report are a few suggestions for creating a sense for size and amount of things, together with practical ideas for the classroom.

Limber up your brain - rough estimations based on mental calculations

1. How high would a barometer be containing water instead of mercury?

Answer: Approximately 10 metres.

Solution: Consider that the working principle of a barometer is based on the balance between the atmospheric pressure (equivalent to the normal atmospheric pressure $p_n = 10^5\,\mathrm{Pa}$) and hydrostatic pressure $p_h = h \cdot \rho \cdot g$ of the liquid (water in or case). If you introduced the density of water and the Earth's gravitational acceleration to the equation $p_\mathrm{n} = p_\mathrm{h}$, the calculated height of the water column (difference in liquid level) would be 10 m.

2. Imagine that somebody would like to balance you out with gold. How much of gold would you need?

Answer: Units of litres.

Solution: Let us imagine a man with about 70 kilos of mass. The human body is mostly composed of water and because water density is approximately $\rho = 1000\,\mathrm{kg \cdot m^{-3}}$, 1 kg of water occupies a volume of 1 litre; thus the man weighing 70 kg occupies a volume of 70 litres. Even if we do not remember the density of gold, we can guess that it is about 1 order of magnitude higher than the density of water (to be precise, the nominal value is $\rho_{Au} = 19290\,\mathrm{kg \cdot m^{-3}}$ [1]). With respect to our rough estimation, the volume of gold which is necessary for balancing out the weight of our body would be ten times lower than the volume of the water we are composed of, i.e. several litres.

3. Estimate the mass of one million steel balls with a diameter of 1 mm.

Answer: Units of kilograms

Solution: Let us use the relation between the mass, density and volume $m = \rho \cdot V$. The density of steel is estimated to be $\rho \approx 5000\,\mathrm{kg \cdot m^{-3}}$ (higher than the density of water but lower than the density of gold). The values of real density of different types of steel range from $7500\,\mathrm{kg \cdot m^{-3}}$ to $8300\,\mathrm{kg \cdot m^{-3}}$ for different kinds of steel, but our simple estimations are accurate enough for our purposes. The volume of one ball is

$$V = \frac{4}{3}\pi \cdot r^3 \approx \frac{4}{3} \cdot 3 \cdot \left(\frac{1}{2}\right)^3 \mathrm{mm^3} \approx \frac{1}{2}\mathrm{mm^3}$$

(one student estimated it considering the simple idea of a ball trapped in the cube with 1mm edge. Because the ball did not fill up the space completely, we considered just half of the volume or $\frac{1}{2}\mathrm{mm^3}$). One million steel balls would occupy a volume a million orders of magnitude bigger, $\frac{1}{2}\mathrm{dm^3}$; multiplying it by our estimated steel density, the resulting mass is 2 to 3kilograms.

4. How many moles of water do you carry on your back?

Answer: Units of thousands of moles, i.e., kilo-moles.

Solution: The mass of one mole or the molar weight of water is equal to $M_m = M_r(H_2O)\,\mathrm{g \cdot mol^{-1}} = [2 \cdot A_r(H) + A_r(O)]\,\mathrm{g \cdot mol^{-1}} \approx (2 \cdot 1 + 16)\,\mathrm{g \cdot mol^{-1}} \approx 18\,\mathrm{g \cdot mol^{-1}}$. One thousand moles have a mass of about 18 kg which is the mass of a heavier bag. We are able to carry approximately thousands moles of water, i.e. units of kilo-moles.

5. Which one contains more money- one wagon of 10-crown coins or ten wagons of one crown coins?

Answer: Ten wagons of crown coins

Solution: We can start with following idea: ten wagons of 1 crown coins give the same amount of money as the 1 wagon of 10-cown coins, assuming both coins have the same volume. A 1 crown coin is smaller than a 10 crown coin, thus more of them can be put into a wagon, so 10 wagons filled with 1 crown coins contains a bigger financial deposit than 1 wagon filled with 10 crown coins.

Have you ever seen one million or do you have a sense about this huge amount?

This activity could help the students to create an impression of huge amounts. It is suitable for children of different ages. It is appropriate to separate children into several groups containing 6 to 8 members, because they will stick together a huge number of millimetre paper arches. The activity has two variants but the beginning of both is still the same.

Task: Stick together so many arches of paper that the number of millimetre squares in the resulting structure approaches 1 million. Mark then so many squares that the amount

a) is equal to the number of the citizens in your regional capital, local town or the village you live in

b) is the number of hours that a 100 year old person is alive, or the number of hours you spend in a school

Accessories: a lot of millimetre arches (20 arches per group), scissors, glue, ruler, calculator, stationery

Procedure: Using a simple calculation, students will discover that to reach 1 million of millimetre squares, approximately 20 (16) arches of A4 paper are needed (the size of final arch will be equal to 1m^2).

a) To estimate the number of citizens that live in the regional capital, students at first estimate the number and then find more precise data on the internet. Then they mark the area which contains the relevant number of millimetre squares by line. It is appropriate to suggest choosing a square area-it is the clearest way of demonstration. If it is possible, have them mark the area representing the town (village) where they live. If the students from different groups live in different regions, each of them can mark their own town (village). The procedure is similar when emphasizing the number of classmates. The only problematic regional capital is Prague, having 1200000 inhabitants, too large to fit on the millimetre paper. In that case, we consider the whole paper to be Prague.

b) Students will calculate how many hours would fit into 100 years. Considering that the year has 365 days on average and the day contains 24 hours, the result would be $n = 100 \cdot 365 \cdot 24 = 876000$. A one hundred year old person will be alive for 900 000 hours which could seem to be not enough! Marking of the appropriate values (hours in that case) could be done the same manner as in the previous case.

How many hours will we spend in grammar school? Each lesson has 45 min, but adding the breaks and the time for lunch in a canteen, we can count 60 min per lesson, because we are interested only in rough estimation. Assume that we haven't croaked, the time we spent in grammar school reaches 9 years. Every school year has 33 weeks and every week contains 25 lessons. The number of hours we spend in a grammar school is then $p = 9 \cdot 33 \cdot 25 = 7425$. It is approximately 1% of 100 years and more than 1% of average age which is not so much. The students would probably not agree.

Methodological remarks: At first, the teacher should figure out whether to let the students use a calculator or not. If you allow a calculator, I suggest that they use those in their mobile phones, even if it is not commonly allowed. They will practise the work with the machine they carry all the time. In variant b) it is possible to count and mark how many hours they usually spend at work. It is good to discuss all the acquired data together with the students and it is appropriate to let them subjectively sum up if the particular numbers seem to be huge or not.

How long is our corridor or measurements leading to more precise estimations.

This activity is suitable as a slightly nontraditional introduction to measurements, especially for younger students. It uses no special equipment and is suitable for work in pairs.

Task: Try to estimate the length of a school corridor

a) Estimate the length standing at one of its ends

b) Estimate the length walking along the corridor as fast as you usually walk, without stopping

c) Measure the length using objects which do not have a scale in units of length (you are not allowed to use a ruler)

Tools: stationery, all the instruments that fulfil c).

Procedure:

a) Students will either guess how long the corridor is or they will base their estimation of its length on the distance between two neighbouring class-doors, width of the window, distances between the lines on the floor etc. The estimated values will be marked carefully.

b) Working on this problem, they probably will not take into account the possibility of counting the number of steps and multiplying it by the length of their foot (or the step itself). The resulting values should be precisely marked again.

c) This task forces the students to take some object with known length, such as a notebook, or different parts of the human body (such as the arms). Some students will enthusiastically lie on the floor to measure the length of the corridor using their own body.

Methodological remarks:

If we compare the methods described in a, b and c, we can guess that the least precise is method a, more precise is method b and the most precise one is c. It happens sometimes, that some pairs reach more accurate results from estimations than from the measurements. It could happen by good fortune or be caused by some systematic error, as when the length of the notebook is measured incorrectly. If we calculate the arithmetic average of the results of all groups separately for the methods in a, b and c it is highly probable, that c would be more accurate than a and b. Considering the differences in estimations and direct measurements in a, b and c, the differences may not be dramatic.

Which one is heavier or "how the objects lie".

Fig. 1: Which is heavier? Do not be misled by the size and the material.

This activity should directly explain, that the measuring the size of objects by pure balancing in the hand is unreliable, because we are misled by our own experience (like assuming that big or metal objects must be heavy).

Task: Compare two objects a) the bigger one which is lighter than the smaller one, b) the bigger one is really heavier than the smaller one, keeping each in a different hand at the same time.

Tools: heavy small object and light bigger object-a chemical frame and an aluminum ladder, heavy big object and small lighter object-plastic bottle filled with water and a letter weight, weights (personal and kitchen)

Procedure: Prepare at first the couple of objects where the bigger one is heavier than the smaller one. The frame or the ladder is appropriate.

We will let the students hold both objects in their hand and ask them to not describe or somehow comment the situation, only to mark personally which one seems to be heavier. After finishing this experiment with all students, we will compare all estimations and weigh both objects precisely. After the discussion, we will start comparing the second couple of objects.

Methodological remarks: A lot of students will consider the ladder to be heavier than the frame. To persuade them of the real truth, weigh both of them on a personal scale. Asking the students why they judged the bigger object to be heavier, the resulting answer would be that it was due to its size. Having the new experience that the bigger object can be lighter than the smaller one now, some of the students will use it in comparing the mass of heavier and bigger objects. Discussing afterwards why most of them have been misled, it is good to point out the fact that we usually consider the metal object to be the heavy one. Thus the main result is that the comparison of the weight of different objects only by balancing it in a hand is deceptive.

Conclusion: Without any calculator and even any stationery, it is possible to estimate the values of different qualities only using simple estimations. The idea of huge quantities can be visualised by using the millimeter paper. A measurement is usually more precise than a rough estimation, but when trying to guess approximate values, we can use a comparison with the value of a known quantity; thus the difference between the measurement and the estimation might not necessarily be so dramatic, and thus the judgement is not without value. On the other hand, it is important to keep in mind that estimation could be strongly influenced by conditions that could not be totally ignored (like size of the object or the material, estimating its weight and comparing it with another object).

References:

[1] Mikulčák J. a kol.: *Matematické, fyzikální a chemické tabulky pro střední školy.* Prometheus Praha, 1988. ISBN 80-85849-84-4.